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THE

STANDARD ARITHMETIC,

FOR

SCHOOLS OF ALL GRADES

AND FOR

BUSINESS PURPOSES.

In Two Numbers.

NUMBER ONE.

BY

JAMES E. RYAN,

PRINCIPAL OF SCHOOL NO. 26, BROOKLYN.

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PREFACE.

In the preparation of this work the author has sought to make a practical and properly-graded book.

It is believed that the exercises are well adapted to illustrate the theory as developed in the successive articles, and that the number and scope of the problems will fully satisfy the requirements of thorough instruction, without necessitating the repetition of solutions, and thus making the work of pupils monotonous and irksome.

Considerable labor has been expended in the preparation of problems which should prove useful in other departments of science.

The matter of oral work has not been neglected. The analyses and problems are such that an extra text-book in Mental Arithmetic will be superfluous. The processes applicable to small numbers are logically similar to those required for large ones. The analyses of problems requiring two or more operations are intended for daily use; and, if these formulæ are customarily stated at recitations, this Written Arithmetic can hardly fail as a means of intellectual discipline.

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The publication of an arithmetic in two volumes has these recommendations: Each volume is light and inexpensive. When the second volume is taken up, it will be found clean and perfect, and will, moreover, be novel, and consequently attractive.

In each of a few recent treatises, the "First Part" is devoted solely to the abstract nature of arithmetical operations, the application of the theory being reserved for the "Second Part," termed "Business Arithmetic." In this work, the practical use of each operation follows the unfolding of its fundamental principles, the author being fully convinced that not only do theory and practice mutually aid each other, but that few youths can afford to spend much time on mere theory and wait for the *chances* of learning "Business Arithmetic" at a later period.

The author is indebted to the works of Haswell, Tracy, Stoddard, Colburn, Greenleaf, Felter, French, Robinson, and Willson, American authors; and to those of Hensley, Dalton, Cornell, Fitch, Galbraith, and Barnard, all British authors of repute.

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SUGGESTIONS TO TEACHERS.

MAKE sure that every definition and principle is clearly comprehended. Brevity is not always conducive to simplicity. Teachers will do well to expand these condensed statements, using even homely illustrations. When *understood*, the definitions and principles should be *memorized.*

Encourage pupils to question, to dispute, modestly and politely, what does not seem reasonable. Require them *to state plainly what needs to be done* or *what has been done*, and, above all, to prove every result.

A pupil will gain far more from the thorough performance of simple work than from attempts at the solution of abstruse problems before he has laid a proper foundation for such work.

Do not give direct aid; rather suggest such easier problems as will lead the learner gently upward. Make every failure an occasion for searching questions on what has been previously studied and explained, in order to secure a clearer comprehension of the principles involved.

See to it that the pupils are well drilled in the application of the analyses. Do not take it for granted that, because you can state all the processes in regular sequence, the matter is equally clear to the members of your class. Teach them to state the work to be done before they begin to perform the fundamental operations required.

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Direct the attention of your class to the intimate relations of the different operations, and endeavor to make them realize how few and simple the principles are on which the various rules are founded. In such a chain as that made up of Division, Common Fractions, Decimals, and Percentage, whenever a link is struck, the whole should vibrate.

Cultivate habits of neatness and order. Slovenly work should never be accepted.

BE EXPLICIT. In assigning lessons let there be no doubt as respects what is to be learned. In hearing recitations be plain and fair.

BE WATCHFUL. Demand undivided attention. A teacher who understands his subject can, by the exercise of his own will, control that of his pupils. The body of every lesson should be recited by the feebler pupils, while the abler members of the class are kept alert by appeals to their memory and judgment as referees and auxiliaries.

BE BRIEF. Brevity should never be forgotten in anecdotes, illustrations, and solutions. Pupils should be prompt and concise. The use of the words *plus*, *minus*, sum, difference, and times, and such phrases as "one-third of six," "contained in," "per cent.," should be enjoined, instead of their more extended equivalents.

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ARITHMETIC.

Section I. Definitions.

1. **ARITHMETIC** treats of the nature and practical use of numbers.

2. A **number** is that which may be counted and consists of one or more units. It answers the question, How many?

3. A **unit** is a single thing or one. It is anything considered as a whole—that is, without reference to its parts.

4. A number which refers to some object named is called a **concrete number**. Thus, in the phrase "six cents," *six* is a concrete number.

5. A number used without special reference to any object, is a **pure** or **abstract number**; as five, twenty.

6. Numbers may be expressed by **objects**, by **words**, by **letters**, and by **figures**.

7. The fingers will indicate from one to ten. By the Roman letters I, V, X, L, C, D, and M any number may be readily shown. The Arabic figures 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0 are the most convenient means of expressing numbers.

8. **Numeration** is the art of expressing numbers in words that are already written in figures.

9. To name numbers there are required but fifteen distinct words; viz., one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, hundred, thousand, million.

10. Axiom I. **All the parts of a number make up the whole.**

11. Two is one and one. Three is one and one and one, or

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two and one. Four is three and one. Ten is nine and one. Eleven is ten and one. One hundred is ten tens.

12. Three and ten make thirteen. We do not think or say "nine and four," or "seven and six," but *three* and *ten*.

13. Numbers are considered as made up of **tens**. Ten being the basis of our system, it is called a **decimal system.** *Decem* (Latin) means ten; hence our word decimal. Thirty-five equals three tens and five; twenty-four, two tens and four; one hundred is ten tens; two hundred and fifty is two tens of tens and five tens of units.

14. ***Notation*** is the art of *writing* numbers in characters.

15. *Note.*—The numbers of chapters and other divisions of books and of documents are usually written by means of **letters** as follows:

I	V	X	L	C	D	M
one	five	ten	fifty	one hundred	five hundred	thousand

After having learned the Arabic notation, pupils can acquire a thorough knowledge of the Roman method with great ease. We shall therefore defer its discussion till later.

Section II. Arabic Notation.

16. In all arithmetical computations, numbers are expressed by the **Arabic system of Notation.**

17. This system employs ten figures to represent numbers, viz.:

0	1	2	3	4	5	6	7	8	9
naught	one	two	three	four	five	six	seven	eight	nine.

18. Naught, 0, expresses no value by itself, but serves to **locate** the other figures.

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19. The last nine above mentioned are called **significant figures.** They are also called ***digits.***

Note.—The numerals are called **digits** from *digitus* (Latin), a finger.

20. ***Position.*** The **value** of each **digit** is shown by its **position.**

A digit standing alone expresses ones, or units. Thus, 6 means six units.

21. A figure on the left of a single figure is said to stand in the **second order.** If it be a significant figure, it stands for one or more ***tens.***

22. A figure standing in the third place from the right is of the **third order.** Digits of the third order denote **hundreds.**

23. Accuracy in **notation** is of the **highest importance.** If the figures employed are not written correctly, the labor performed is wasted. "Advanced" classes are not unfrequently found deficient in this the alphabet of arithmetic. Teachers having young classes are recommended to present the following simple and practical scheme of notation. It is based on a suggestion made by Dr. Wickersham.

24. Arrange columns of figures on a card or blackboard thus :

1,000,000	100,000	10,000	1,000	100	10	1
2,000,000	200,000	20,000	2,000	200	20	2
3,000,000	300,000	30,000	3,000	300	30	3
4,000,000	400,000	40,000	4,000	400	40	4
5,000,000	500,000	50,000	5,000	500	50	5
6,000,000	600,000	60,000	6,000	600	60	6
7,000,000	700,000	70,000	7,000	700	70	7
8,000,000	800,000	80,000	8,000	800	80	8
9,000,000	900,000	90,000	9,000	900	90	9

Use the first two columns for the first lesson. Call the first the *units* column, and the second the *tens* column. Ask the

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pupils to notice that, up to 9, there is a figure for *each number*. After having arranged objects, as pins, strokes on a blackboard, etc., in groups of ten, and having called on the class to count them, thus, "one ten, two tens, three tens," then show that 1 ten is written with the figure 1 at the left of a cipher, thus occupying the second place.* Two tens is shown by the figure 2 on the left of a cipher, etc. Exercise the class in pointing out 2 tens, or 20; 5 tens, or 50; 7 tens, or 70. Afterwards cause them to write from dictation "two," "seven," "nine," "two tens," "seven tens," etc.

At the second lesson, the teacher should review the previous work and then proceed to ask questions like these: How many added or put with 10 will make 11; how many with 10 will make 12; 13; 14; 17; 19? Then ask how these sums or numbers must be written. If answered, or if not, erase the cipher and (if the number be 11) write 1 in its place, saying that the right hand order is the place of units and the second order is the place of tens. For 14 show that the 4 units must take the place of the cipher and that the ten is already written, and so go on substituting up to 20. Show by counting groups of objects, or by means of blackboard dots, that 2 tens make 20, and point out the number as written. Proceed patiently with the numbers between 20 and 30, etc., to 99.

25. Ask questions as follows: What number is made up of 1 ten and 7 units? What number, of 2 tens and 4 units; 4 tens and 3 units; 7 tens and 8 units; 8 tens and 5 units; 3 tens and 6 units, etc.? How many tens and units in twenty-three? In seventy-six? In seventeen? In eighty-two?

To introduce the subject of hundreds, make a simple diagram by means of dots on a blackboard, ten columns of ten points each; contrast it with a column or row of ten points, and also with one point; and, after stating that the last is written with

* In Arithmetic we proceed, in this respect, in a direction opposite to that which we pursue in reading. The Arabs read from *right* to *left*, and *from below, upward*.

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1, and the next before with 10, say that the large group is indicated by the figure 1 written on the left of *two* ciphers; in other words, in the third place or order. Then point to the successive numbers of hundreds as the pupils read, and ask questions like those used in the preceding lessons. A thorough drill in writing and analyzing quantities in three figures will lay a reliable foundation for instruction in thousands and ten thousands.

After a few lessons by the foregoing method, the whole matter should be reviewed and restated as follows:

26. Each figure standing alone expresses *ones* or *units*.

27. When more than *nine* are to be expressed, two or more figures are required, and are placed side by side. That on the right signifies *units;* the next figure counts for *tens;* the next, for *tens* of *tens* (hundreds); the next, for tens of tens of tens (thousands); and so on.

Note.—A *simple unit* is a single thing. A *collective unit* denotes a group of ones considered as a whole. *A number* is a collection of units of the same kind.

Orders.

28. Numbers are made up of orders. The *first order* is that on the right and indicates ones; the second order expresses tens; the third order, hundreds; the fourth order, thousands, etc.

29. (*d*) A group of ten ones makes a unit of the second order and is called a ten; if there be two such groups, two tens, etc. A group of ten tens is called a hundred. A group of ten hundreds is called a thousand.

30. When a figure stands on the left of another similar figure, it expresses ten times as much as that on the right. In general

31. Orders increase from right to left by the scale of ten.

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TABLE.

Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
6	6,	6	6	6,	6	6	6,	6	6	6

(*a*) The first six on the right hand means six units; the second, having one figure on its right, is of the second order, and means six tens; the third stands for six hundreds; the fourth, for six thousands; the fifth, for six tens of thousands; the sixth, for six hundreds of thousands; the seventh, for six millions; etc.

Exercise I.

Underline all the figures in the following groups which mean:

1. Hundreds, 426; 9342; 89716; 345; 14296; 791; 184269; 345867; 245163.
2. Tens, 39; 42; 964; 4285; 38; 391; 423; 787.
3. Thousands, 49673; 3429; 814445; 2467111; 3429; 84963; 33345; 8142256; 774935; 2461.
4. Millions, 3975426; 84973214; 1000000; 20000000.

32. The **cipher** standing alone has no value; it means *nothing*. As a **locater**, it *places* the other figures. When a number is read or numerated, the order occupied by a *cipher* is not mentioned; thus, 45063 is read, forty-five thousand and sixty-three: it must, however, *never be left unwritten*. We have no special names for the fifth, sixth, and the following places, the orders being named by combining the words ten and hundred with other names.

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Periods.

33. Large numbers are separated by commas into **periods of three figures** each. We then have *Units*, *Tens*, and *Hundreds* of each period. The names of the periods, or grades, counting from the right, are as follows: Units, Thousands, Millions, Billions, Trillions, Quadrillions, Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, etc.

Periods, .	5th			4th			3d			2d			1st		
Orders, .	15,	14,	13,	12,	11,	10,	9,	8,	7,	6,	5,	4,	3,	2,	1
Names, .	Trillions.			Billions.			Millions.			Thousands.			Units.		
	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.
Number,	2	2	2	2	2	0	0	2	2	2	0	2	2	2	2

This number is read: Two hundred and twenty-two trillions, two hundred and twenty billions, twenty-two millions, two hundred and two thousand, two hundred and twenty-two.

34. The word **units** is usually *understood;* but tens and hundred (tens abbreviated to *ty*) are always heard when represented by a digit.

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NAUGHTS TO BE WRITTEN IN ORDERS NOT READ.

35. Prepare a diagram like the following:

	Trill.	Bill.	Mill.	Thous.	Units.
	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
e.	2	4	1	6	5 0
f.	1 8 2	2 5	3 2	1 5	1 2
g.	4 4	3 2	6 9	4 0	8
h.	4 1 4		2 3		7 9 1
i.	1		4		6 2 3
j.	2 9			4 1	8 4
k.	2 1 6		3 9	8	4 2

Respecting the line of figures marked *i*, it may be said that, as it stands, 14623 only is fairly represented. 1 standing on the left of no more than four figures can hardly be counted as a fifteenth figure. The mere separation of things cannot increase their number. Hundreds of trillions should really stand in the 15th order; so, guided by the headings, we write 2 naughts in the 5th period, 3 naughts in the 4th, 2 naughts in the 3d, and 3 naughts in the 2d period.

36. QUESTIONS. Are the periods of *e* complete? Tell what is required in the first period. What place is vacant in the second period? What other place? etc., etc. How many Units in 7 tens and 3? How many tens in 170, in 230, in 460? How many Hundreds in 90 tens, in 130 tens, in 7000 units, in 10000 units, in 1400 tens? How many Tens in 1000, in 20000, in 1500? How many Thousands in 1500 tens, in 250 hundreds, in 1 million? How many Hundreds in 1 thousand, in 1 million? How many Millions in a thousand thousands, in 5 thousand thousands, in 50 thousand thousands?

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How many millions make a billion? How many thousands? How many hundreds? What is the name of the 10th order, of the 13th, of the 4th, of the 7th, of the 2d, of the 12th, of the 9th, of the 6th, of the 15th, of the 14th, of the 5th? etc.

Exercise II.

Write out in words the separate value of every figure in the following:

Example.—39354; thirty thousand, nine thousand, three hundred, fifty and four.

621; 4,264; 82,529; 31; 32,724,896; 8,004,001; 705; 1,803,040; 44,444; 2,220,022; 13; 11,300,013; 249,249.

Exercise III.

Point off the following into periods, and read the numbers:

20202202; 4441011012401; 3072; 89614; 2530; 848484848; 5004004005; 9609609603; 84482231; 10121012101239; 6802531971495; 5096324; 808000, 41222412220· 350530535; 2999999; 811111146.

Exercise IV.

Write the following in numerals:

Five hundred; five hundred thousand; two millions one thousand and two; twenty-four thousand five hundred and one; ten millions ten thousand and ten; two hundred and fifty-two billions two hundred thousand and two; eighteen millions and sixteen; nine hundred and seventy thousand and nine hundred; thirteen thousand seven hundred and eighty-one; twelve hundred and nineteen; six hundred and six.

37. *Rule for Notation.*

Begin at the left hand with the highest denomination and write out each period as required, filling up the vacant orders or periods with ciphers.

See Exercise IV. and 33.

Note.—A *Rule* tells how to do what is required.

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38. *Rule for Numeration.*

To read large numbers, separate the figures into periods of three figures each, counting from units; then, commencing at the left, read the figures of each period in connection with its name. The ciphers are not to be read.

The name of the right-hand period need not be pronounced. Thus, 4216303401 is pointed 4,216,303,401, and is read four billions, two hundred and sixteen millions, three hundred and three thousand, four hundred and one.

See Exercises II. and III.

39. *Principles of Notation and Numeration.*

1. *Each removal of a figure one place toward the left increases its value ten times.*

2. *The value of each separate figure is shown by its place or order.*

Note.—A **Principle** is an essential part of a subject, or a leading thought in any statement.

40. *Questions for Examination.*

(*a*) What is a Unit? (*b*) What is a Number? (*c*) What is a Quantity? (*d*) What is an Abstract number? (*e*) What is a Concrete number? (*f*) What may express numbers? (*g*) Define Arithmetic. (*h*) Define Notation. (*i*) Define Numeration. (*j*) What *letters* are employed in Roman Notation? (*k*) What are digits? (*l*) What does a figure standing alone express? (*m*) What does the first *Order* in numbers express, and where is it written? (*n*) What does the second order express? (*o*) What are the names of the 1st, 2d, 3d, 4th, 5th, 9th, 3d, 7th, 10, and 8th orders? (*p*) Why is this system called a *Decimal* system? (*q*) Which way do orders of figures increase, and how many times

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for each place? (*r*) What are Periods and what are the names of the first five? (*s*) What are the *two principles* of Notation and Numeration? (*t*) What is the *rule* for Notation? (*u*) For Numeration? (*v*) What is the use of the Cipher? (18) (*w*) What is a Rule? (*x*) What is a Principle?

Section III. Roman Notation.

41. Notation by the *Roman method* employs seven capital letters; viz., I for *one*; V for *five*; X for *ten*; L for *fifty*; C for a *hundred*; D for *five hundred*; M for a *thousand*.

TABLE.

I.	stands for	One.	XV.	stands for	Fifteen.
II.	"	Two.	XVI.	"	Sixteen.
III.	"	Three.	XVII.	"	Seventeen.
IV.	"	Four.	XVIII	"	Eighteen.
V.	"	Five.	XIX.	"	Nineteen.
VI.	"	Six.	XX.	"	Twenty.
VII.	"	Seven.	XXI.	"	Twenty-one.
VIII.	"	Eight.	L.	"	Fifty.
IX.	"	Nine.	C.	"	One hundred.
X.	"	Ten.	D.	"	Five hundred.
XI.	"	Eleven.	DC.	"	Six hundred.
XII.	"	Twelve.	M.	"	One thousand.
XIII.	"	Thirteen.	MM.	"	Two thousand.
XIV.	"	Fourteen.	$\overline{\text{M}}$.		One thousand thous'd.

42. A letter placed *before* another of greater value takes its own value from the greater, as IV; but if it comes *after* a greater,

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its value is added, as XV. (*See the table.*) *Repeating* a letter repeats its value. A *line* drawn *over a letter* increases its value a thousand times.

Exercise V.

Read I, IV, VI, X, XI, IX, XII, XIII, XVI, XX, XIX, XXI, XXVII, XXX, XL, LX, LXX, XXXIX, C, CC, CCC, CD, $\overline{C}$, $\overline{L}$, M, MD, MDCCC, LXXVI, MDCCCLXXVII, MCDXCII, $\overline{I}$, $\overline{V}$, $\overline{V}$ICCXXIII.

Exercise VI.

Write, in the Roman Notation, 42 27 6 19 29 51 500 2000 1876 1877 1492 1620 38 97 65 65 83 1900 1944 1871 375 555 426

ADDITION.

Section IV. Oral Exercise.

43. How many pins are 2 pins and 3 pins? If 2 peaches lie in a basket and a person should then put in three other peaches, how many would there be in the basket? A boy owns 3 rabbits, 2 dogs, and a kitten; how many animals has he? If a girl has 2 apples and 7 lemons, how many has she? How many what? Answer, "Fruits." Can she be said to have 9 apples, or 9 lemons? When can one number be equal to two other numbers? When numbers are alike, or can be made alike, what are they

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made up of? Answer, "Common units." How much are two dimes and 2 cents? Answer, 2 dimes make 20 cents; with these count 2 cents, and the sum is 22 cents. How much is 1 quart and 1 pint? 1 quart makes 2 pints, to which if we add the 1 pint mentioned, the sum or amount will be 3 pints.

44. Addition is the process of finding a number which will exactly equal two or more other numbers taken together.

Note.—*Like* numbers only can be added.

45. The sign of Addition + is called **plus.**

46. The numbers to be added are called **addends.**

47. The result is called the **sum.** It is set opposite to the numbers added, and is separated from them by two level parallel lines. These lines, =, are called the **sign of equality.**

48. A question in Addition, with the answer thereto, may be written thus: 4+5=9. This expression is read, "Four plus five equals nine."

49. *Note.*—Ordinary counting is, in fact, the addition of unit after unit; for 1+1=2, 2+1=3, 3+1=4, etc. In teaching pupils to count by 2s allow them, *at first*, to say 2 and 2 are 4, 4 and 2 are 6, 6 and 2 are 8, 8 and 2 are 10, 10 and 2 are 12, etc. *Afterward*, phrase it thus: "2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22," etc., repeating the summation many times, commencing now at 6, then at ten, then at 18, etc. When the pupils have acquired facility with *even* numbers, begin with 1, 3, 5, 7, etc., and progress thus: 3, 5, 7, 9, 11, 13, etc., as far as 25. Then count with 3s in the same manner, being careful to make every member of the class thorough in the initial steps. With *young* pupils use the abacus, or convenient objects, as corn or beans. All oral exercises should be very brief. The words should be spoken with decision.

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50. ADDITION TABLE.

0 and 1 are 1; 0 and 2 are 2; 0 and any number make that number.
1 and 0 are 1; 2 and 0 are 2; any number and 0 make that number.

1 and	2 and	3 and	4 and	5 and
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6
2 are 3	2 are 4	2 are 5	2 are 6	2 are 7
3 are 4	3 are 5	3 are 6	3 are 7	3 are 8
4 are 5	4 are 6	4 are 7	4 are 8	4 are 9
5 are 6	5 are 7	5 are 8	5 are 9	5 are 10
6 are 7	6 are 8	6 are 9	6 are 10	6 are 11
7 are 8	7 are 9	7 are 10	7 are 11	7 are 12
8 are 9	8 are 10	8 are 11	8 are 12	8 are 13
9 are 10	9 are 11	9 are 12	9 are 13	9 are 14
10 are 11	10 are 12	10 are 13	10 are 14	10 are 15
6 and	**7 and**	**8 and**	**9 and**	**10 and**
1 are 7	1 are 8	1 are 9	1 are 10	1 are 11
2 are 8	2 are 9	2 are 10	2 are 11	2 are 12
3 are 9	3 are 10	3 are 11	3 are 12	3 are 13
4 are 10	4 are 11	4 are 12	4 are 13	4 are 14
5 are 11	5 are 12	5 are 13	5 are 14	5 are 15
6 are 12	6 are 13	6 are 14	6 are 15	6 are 16
7 are 13	7 are 14	7 are 15	7 are 16	7 are 17
8 are 14	8 are 15	8 are 16	8 are 17	8 are 18
9 are 15	9 are 16	9 are 17	9 are 18	9 are 19
10 are 16	10 are 17	10 are 18	10 are 19	10 are 20

Exercise VII.

How many are 7 and 6? 6 and 7? 17 and 6? 16 and 7? 6 and 27? 6 and 47? 57 and 6? 8 and 5? 5 and 8?

How many are 5 and 7? 85 and 7? 87 and 5? 7 and 85? 5 and 87? 4 and 8? 4 and 38? 38+1+3? 4+2? 104+2? 164+2? 174+2? 274+2? 92+4? 402+4?

How many are 9 and 3? 29 and 3? 8 and 2? 48 and 2? 108 and 2? 102 and 8? 8 and 3? 38 and 3? 3 and 78?

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178 and 3? 278 and 3? 11 and 1? 15 and 2? 10 and 2? 20 and 3? 5 and 30? 6 and 3? 23 and 6?

Add 9 and 7. 9 and 9. 9 and 8. 9 and 57. 9 and 59. 9 and 58. 9 and 47. 9 and 5. 4, 5, and 9. 5, 4, and 19.

Add 9 and 2. 6, 3, and 2. 89 and 2. 2 and 29. 7, 2, and 1. 49 and 1. 51, 1, and 8. 9 and 161. 1 and 179.

What does 6+6 equal? 6+106? 126+6? 226+6? 6+36? 46+3+3? 56+4+2? 5+1+46?

What does 4+6 equal? 104+6? 24+6? 6+14? 5+6? 85+6? 5+76? 9+4? 9+44?

What is the sum of 10 and 10? 10 and 20? 10 and 30? 4, 6, and 70? 7, 3, and 80? 10 and 5? 4, 5, and 8? 3, 6, and 19? 4, 2, and 7? 7, 3, and 2? 8, 4, and 8? 21, 5, and 3? 93, 4, and 3?

Count by 4s up to 100, commencing with 4, 5, 6, or 7; thus: 4, 8, 12, 16, etc.; 5, 9, 13, etc.; 6, 10, 14, etc.; 7, 11, 15, etc. Afterward count by 5s, 6s, 7s, 8s, 9s, and 10s.

51. Add these columns many times. "Slow and Sure" does not apply in Addition. It should read "**Active** and **Accurate,**" or "**Rapid** and **Right.**"

EXERCISE VIII.

3	2	3	5	2	3	4	3	2	1	2
2	2	4	6	7	7	3	2	4	7	3
3	3	5	6	6	7	3	7	3	7	4
1	4	3	2	6	2	7	2	8	7	5
2	2	2	3	4	4	2	8	7	8	6
4	3	5	3	3	7	8	3	3	8	7
3	3	4	5	4	3	3	7	8	8	8
2	4	3	4	5	4	7	2	1	9	9
1	2	4	6	3	7	8	5	9	9	3
2	3	2	2	2	7	2	3	2	9	5
—	—	—	—	—	—	—	—	—	—	—
									73	52

Add 24 and 35.

Five units and four units are nine units; two tens and

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three tens are five tens; five tens and nine units make fifty-nine.

52. Axiom II. **If we add all the parts of several numbers, we add the numbers.** Thus, to add 16 to another number, we may add ten and then add six; or, we may add six and then add ten; or, we may add two and ten and four—*e. g.:* 16+18=? 18 and 2 are 20; 10 and 20 are 30; 30+4 are 34. Therefore 16+18=34.

(1) Add 2 quarts 1 pint and 3 gills, 1 pint and 2 gills, 5 quarts, 2 pints, 3 pints, 4 gills, 7 gills, 3 quarts, 5 quarts, 1 pint, and 1 gill.

Let us write these concrete numbers so that like quantities will stand together, and add each kind separately—*e. g.:*

2 qts.	1 pint.	3 gills.
5 "	1 "	2 "
3 "	2 "	4 "
5 "	3 "	7 "
	1 "	1 "
15 "	8 "	17 "

(2) Let us add, £12 8*s.* 6*d.* £3 4*s.* 3*d.* £1 18*s.* 2*d.*

£	*s.*	*d.*
12	8	6
3	4	3
1	18	2
16	30	11

53. Axiom III. **Concrete numbers to be added must refer to similar objects, and the collected units must be of the same order.**

(3)

Thous.	Hunds.	Tens.	Units.
2	2	2	4
		1	5
1	0	3	2
3	14	6	0
6	16	12	11

6 thousands, 16 hundreds, 12 tens, and 11 units might be written as one number if the parts were classified; for 16 hundreds equal 1 thousand and 6 hundreds, 12 tens equal 1 hundred and 2 tens, and 11 units equal 1 ten and 1 unit; so that the sum is 7 thousand 7 hundred 3 tens and 1 unit, or 7731.

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54. AXIOM IV. **The value of a quantity is not altered if what is taken from one part is added to another.**

(4)

3	4	6	8
2	0	5	1
3	4	9	7
8	2	3	4
16	10	23	20
17	2	5	0

(5)

8	4	1	9	2
7	6	2	3	5
8	4	2	7	6
9	9	9	9	9
32	23	14	28	22
34	4	7	0	2

(6)

5	8	2	6
3	9	7	0
9	8	2	8
4	6	3	9
21	31	14	23
24	2	6	3

Add the following:

(7)	(8)	(9)
4268	56823	84163
3429	34715	45821
7831	63459	40097
15528	154997	170081

Explanation of Example 7.—The sum of the units is 18, which equals 8 units and 1 ten. Setting down the 8 units under the units, we add the 1 ten to the other tens, and have 12 tens, or 1 hundred and 2 tens. Setting these two tens under the tens, and adding the 1 hundred to the other hundreds, we find their sum to be 15 hundreds. We write 5 of the 15 under the hundreds' column, and add the other 10 hundreds, considered as a thousand, to the column of thousands. The sum of the left-hand column is 15, which is written on the left of that part already set down.

The other problems should be carefully explained, and the class drilled until they understand the whole matter.

55. *Rule for Addition.*

I. *Write the numbers to be added so that figures of the same order shall stand in the same column.*

II. *Begin at the right, and add each column in turn. If the sum of any column be less than* 10, *set it under the*

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column added; but if it exceed 10, *set the right-hand figure under its column, and add the left-hand figure or figures to the sum of the next column.*

III. *Write the whole sum of the last column.*

PROOF.

Add the columns downward.

EXERCISE IX.

1. Add the following numbers: 509; 7126; 1403; 214.
2. What is the sum of 8; 19; 24032; 421; 64002; 873?
3. Required the amount of 824631; 10041; 24036; 4003.
4. 748+30+6025+919+22111+29833= ?
5. 241+1867+286+17742+967+426+3014= ?
6. 62421+90004+202014+88094+725= ?
7. 24698+29475+18562+9090204=9,162,939.
8. Add two hundred and six; seven thousand, nine hundred and ninety; four hundred and twenty thousand, five hundred and seven; and two millions and twenty thousand.
9. 9,614+248,671+3,400,699+1,142= ?
10. 8,200,607+94,555+2,888+4+61,401= ?

In any of the examples, the pupils, if they choose, may divide the numbers into periods when writing them on their slates for adding.

11. Add fourteen thousand; one hundred and two thousand, seven hundred and four; nine thousand, one hundred and twenty; and twelve thousand, two hundred and seven.
12. What is the sum of nine; nineteen; ninety; nine hundred; and nine hundred and ninety-nine thousand, nine hundred and ninety-nine?
13. How many are 856+9193+8765+4287+6696+9185+979? *Ans.* 39,961.
14. 103+125+9+641+10+449+6072 = what?
15. Find the sum of 15 millions 9 thousand and 17; 19 millions 508, 675 thousand 899, and 245 millions 326 thousand and 8. *Ans.* 280,011,432.

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EXERCISE X.

TABLE A.	TABLE B.
1 0 1 1 3 2 5 *x*	4 0 4 4 0 4 3 0 3 *x*
4 2 8 3 5 7 4 9 *w*	2 2 0 0 2 2 0 0 2 2 *w*
7 1 9 3 6 0 6 *v*	9 1 7 9 6 7 9 4 7 *v*
9 4 7 9 3 5 *u*	3 8 4 3 8 8 5 6 *u*
9 8 3 6 4 2 8 2 *t*	1 4 2 3 9 1 8 2 5 4 6 *t*
2 7 3 5 4 4 9 1 *s*	1 9 4 8 5 *s*
1 2 4 0 0 6 5 3 *r*	5 8 9 2 8 4 6 *r*
6 7 5 2 8 3 5 4 8 *q*	1 5 2 4 2 8 3 4 *q*
9 5 4 8 7 9 2 4 3 *p*	8 9 3 6 1 4 8 3 *p*
1 4 0 6 6 7 9 2 2 8 *o*	6 4 2 8 7 8 9 2 4 *o*
9 8 1 2 3 4 5 6 7 3 *n*	3 3 2 8 1 4 2 4 9 3 *n*
8 7 6 5 4 3 2 1 9 8 *m*	1 4 5 9 6 8 4 *m*
1 3 4 2 8 7 6 5 4 3 *l*	2 8 3 6 7 9 5 9 5 *l*
3 2 8 2 8 3 7 6 5 4 *k*	8 3 3 6 7 4 9 4 5 *k*
3 6 1 5 4 *j*	1 2 4 7 7 8 8 6 6 6 *j*
4 0 4 0 4 0 4 *i*	5 1 8 4 0 0 4 7 *i*
1 8 2 4 0 6 7 5 3 *h*	8 3 7 5 0 4 9 8 3 *h*
2 4 0 0 6 4 0 5 *g*	5 8 2 7 6 3 *g*
8 1 9 3 *f*	9 4 5 8 2 1 4 6 *f*
7 2 4 6 5 9 *e*	4 1 1 2 8 9 3 9 3 9 3 *e*
1 9 0 0 0 4 8 *d*	5 9 7 6 8 2 4 5 4 *d*
8 1 9 2 3 7 5 0 4 4 *c*	5 4 2 0 0 4 7 9 6 3 0 *c*
5 1 7 4 2 6 *b*	1 2 4 3 6 4 5 2 8 5 4 *b*
6 4 3 9 2 8 7 *a*	4 9 3 6 8 5 2 4 2 4 *a*

The numbers written under A and B can be combined in many ways, and will afford scores of problems to an ingenious teacher.

The questions which follow are merely introductory and suggestive.

Note.—These numbers may be used for problems in Numeration, Subtraction, Multiplication, Division, Fractions, Denominate Numbers, Square Root, Cube Root, Mensuration, etc. Thus: "Find the number of inches in *t* furlongs, Table A."

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A.		B.	
1.	Req. the sum of the Nos. above *q*.	23.	Ditto.
2.	" " " " *j*.	24.	"
3.	" " " " *f*.	25.	"
4.	" " " " *c*.	26.	"
5.	" " " " *b*.	27.	"
6.	" " " " *s*.	28.	"
7.	" " " " *l*.	29.	"
8.	" " " " *n*.	30.	"
9.	" " " " *v*.	31.	"
10.	Req. sum of *all* the Nos. under A.	32.	"
11.	" " *i j k l m* and *n*.	33.	"
12.	" " *a b c d e f g* and *i*.	34.	"
13.	" " *q r s t u v* and *w*.	35.	"
14.	" " *a c e g i k* and *m*.	36.	"
15.	" " Nos. between *a* and *q*.	37.	"
16.	" " including *k* and *x*.	38.	"
17.	" " " *g* and *r*.	39.	"
18.	" " " *h* and *n*.	40.	"
19.	" " *a d g j m p s* and *v*.	41.	"
20.	" " *b e h k n q t* and *w*.	42.	"
21.	Count from *c* adding every 3d No.	43.	"
22.	" " *b* " " 5th No.	44.	"

Note.—The questions under Table A may be applied to Table B, and, when so applied, are numbered 23, 24, 25, etc.

45. Add together four, five, nine, fifty, three hundred and eight, ten thousand, two thousand and nine, forty-eight, sixteen, and twelve. *Ans.* 12,461.

46. Find the sum of seventy-eight, two hundred and six, four hundred and eighty-three, twelve thousand three hundred, fifty-four millions, two thousand six hundred and seven.

Ans. 54,015,674.

47. What is the total amount of three hundred and five, seven thousand and ninety-six, six hundred and forty-seven,

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eight thousand three hundred and fifty-four, two hundred and eighty-six, four thousand three hundred and eight, seven hundred and forty-five? *Ans.* 21,741.

48. If an exchange broker have in one place 27 gold pieces and 156 silver ones; in another, 758 copper coins, 123 of gold, and 287 of silver; in a third, 96 of each; in a fourth, 37 of gold, 29 of silver, and 100 of copper, how many coins has he, and how many of *each* kind?

Ans. 1. 1,805. *Ans.* 2. Gold, 283; silver, 568; copper, 954.

49–52. (49) How many days in the first 5 months of 1875? (50) In the first 3? (51) The first 10? (52) The first 7?

53, 54. The less of two numbers is 117; their difference is 196. (*a*) What is the greater? (*b*) What is their sum?

56. *Dollars and Cents.*

In writing a number standing for dollars and cents, it is necessary to point off with a period the two orders on the right for cents, and to consider the figures on the left of the period as standing for dollars. Let it be required to write one hundred and eleven dollars and eleven cents—written, $111.11. The sign $ indicates the concrete quantity to be United States money. Dollars, according to law, must be reckoned as units. It takes ten dimes to make a dollar, and ten cents to make a dime. Eleven cents equal one cent and one dime. The figure on the right stands for the cent; that on its left for the dime. The latter means, of course, ten times as much as the former figure. The third figure expresses one dollar; and standing on the left of that representing a dime, indicates ten times as much.

57. Decimals. When anything is separated into ten equal parts, one of the equal parts is called a *tenth;* two of them, two-tenths, etc.

A dime is one-tenth of a dollar, and a cent is one-tenth of a dime. As it takes two halves to make a whole thing or

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unit, and as three thirds, four fourths, and eight eighths are each equal to a unit, so ten tenths will equal a unit. If a tenth be divided into ten equal parts, one of the parts is called a tenth of a tenth, or *a hundredth.* Ten of the small parts will equal one of the larger parts, but it would require a hundred of the smaller parts to make the unit first divided.

TABLE.

10 mills make 1 cent.
10 cents " 1 dime.
10 dimes " 1 dollar.

As dimes and cents are parts of a dollar, which is the unit of money, they are called *fractions;* and because they are tenths, or tenths of tenths, they are called **decimal fractions.**

Note.—If necessary, make a diagram for illustrating the above. A line, properly divided, will serve

Exercise XI.

Find the totals of the following sums of money:

(1)	(2)	(3)	(4)
$1.25	$10.10	$5.05	$4.15
2.10	20.10	4.05	2.25
3.45	30.10	3.05	3.10
6.82	40.10	8.15	4.50

5. John spent 90 cents for a geography, 54 cents for an arithmetic, 8 cents for a slate, 75 cents for an algebra, $1.08 for a reader, and $2.25 for skates, and then had $2.17 left. What had he at first? *Ans.* $7 77.

(6)	(7)	(8)	(9)
$2.46	$0.76	$84.19	$10.25
6.19	1.42	35.12	8.75
10.00	.93	6.43	2.10
8.50	7.42	8.17	4.64
7.20	6.84	6.41	3.95
9.60	3.96	8.25	14.26

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10. How many cents in 14 mills+13 mills+19 mills+17 mills+16 mills+11 mills? *Ans.* 9 cents.

58. The following numbers show the square miles covered by the oceans and the lakes:

(11)		(12)		(13)	
Pacific,	65,630,000	Aral,	26,400	Vic. Nyanza,	28,000
Atlantic,	34,780,000	Baikal,	15,200	Albt. "	26,000
Indian,	30,590,000	Balkash,	6,400	Tanganyika,	13,000
Arctic,	5,930,000	Caspian,	132,000	Nyassa,	8,000
Antarctic,	4,940,000	Dead Sea,	500	Tchad,	15,000
Total,	141,870,000				

(14)		(15)	
Superior,	31,400	Maracaybo,	5,300
Michigan,	25,600	Titicaca,	3,500
Huron,	23,800	Nicaragua,	3,500
Erie,	10,000	Great Salt,	3,200
Great Bear,	9,300	Chapala,	1,350
Winnipeg,	8,900	Tulare,	700
Ontario,	7,300	Moosehead,	300

What are the sums of the heights of the following mountains:

(16) *Cascade and Sierra Nevada.*		(17) *Rocky.*		(18) *European.*	
	Feet.		Feet.		Feet.
Whitney,	14,887	Harvard,	14,384	Elbrooz,	18,526
Shasta,	14,440	Gray,	14,341	Blanc,	15,784
Tyndall,	14,386	Lincoln,	14,297	Rosa,	15,223
Dana,	13,277	Long's,	14,271	Pyrenees,	11,200
Elias,	19,283	Pike's,	14,147	Etna,	10,874

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Find the total population of the following groups, viz.:

POPULATION OF CITIES.

	Illinois.			*Ohio.*	
(19)	Alton,	8,665	(20)	Akron,	10,006
	Aurora,	11,162		Cincinnati,	276,239
	Belleville,	8,146		Cleveland,	92,829
	Bloomington,	14,590		Columbus,	31,274
	Chicago,	298,977		Dayton,	30,473
	Galesburg,	10,158		Hamilton,	11,081
	Peoria,	22,849		Portsmouth,	10,592
	Quincy,	24,052		Sandusky,	13,000
	Springfield,	17,364		Toledo,	31,000
	Pennsylvania.			*New York.*	
(21)	Alleghany City,	53,180	(22)	Albany,	69,422
	Allentown,	13,884		Auburn,	17,225
	Erie,	19,646		Brooklyn,	396,099
	Harrisburg,	23,104		Buffalo,	117,714
	Lancaster,	20,233		New York,	942,292
	Philadelphia,	674,022		Rochester,	62,386
	Pittsburgh,	86,076		Syracuse,	43,051
	Reading,	33,930		Troy,	46,465
	Scranton,	35,092		Utica,	28,804
	Massachusetts.			*Virginia.*	
(23)	Boston,	250,526	(24)	Danville,	3,463
	Cambridge,	39,634		Lynchburg,	6,825
	Fall River,	26,766		Manchester,	2,599
	Lawrence,	28,921		Norfolk,	19,229
	Lowell,	40,928		Petersburg,	18,950
	Lynn,	28,233		Portsmouth,	10,492
	Salem,	24,117		Richmond,	51,038
	Springfield,	26,703		Staunton,	5,120
	Worcester,	41,105		Winchester,	4,477

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(25) *Cities of G. Britain.*		(26) *France.*		(27) *Germany.*	
London,	3,500,	Paris,	2,000,	Hamburg,	350,
Manchester,	520,	Lyons,	325,	Breslau,	180,
Liverpool,	500,	Marseilles,	320,	Dresden,	210,
Birmingham,	350,	Bordeaux,	200,	Berlin,	900,
Leeds,	260,	Lille,	160,	Munich,	175,
Sheffield,	240,	Toulouse,	125,	Cologne,	130,
Bristol,	185,	Nantes,	120,	Magdeburg,	115,
Glasgow,	500,	St. Etienne,	110,	Leipsic,	110,
Edinburgh,	200,	Rouen,	100,	Hanover,	105,
Dublin,	300,	Toulon,	80,		
Belfast,	175,	Havre,	75,	(Thousands)	

(28) *Austria.*		(29) *Italy.*		(30) *Other Europ. Cities.*	
Vienna,	850,	Rome,	200,	St. Petersburg,	700,
Pesth & Buda,	270,	Naples,	400,	Madrid,	350,
Prague,	200,	Milan,	200,	Brussels,	175,
Trieste,	120,	Turin,	200,	Constantinople	1,000,
Lemberg,	90,	Palermo,	200,	Lisbon,	224,
Gratz,	85,	Genoa,	140,	Stockholm,	135,
Brunn,	75,	Venice,	120,	Geneva,	50,
Szegedin,	70,	Florence,	120,	Amsterdam,	270,
Cracow,	60,	Bologna,	110,	Athens,	50,
(Thous.)		(Thous.)		(Thous.)	

Questions to follow Addition.

59. Define Addition. What kind of numbers may be added, and what cannot be expressed as one number? Give illustrations of numbers which cannot be united in one sum. Give the name and form of the sign of Addition. What is the answer to a question in Addition called? What is a general name for an answer? (A Result.) What sign is sometimes placed between the sum and the numbers added? How

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is it formed? How is it read? Give an example of an easy question, together with the answer. Is counting, adding? Give an example of counting by twos, beginning with 2. Count by twos and begin with 5; with 11; with 18. Count by threes beginning with 3; beginning with 4; with 7. Count by sixes beginning with 2; with 5; with 7; with 8. Recite the table beginning 9 and 1. How much are 2+7+13? 4+15+26? Add 24 and 35, and explain the method orally. Repeat Axiom I. Repeat Axiom II. What is a concrete number? What are orders in numbers? What is Axiom III. When there are more than 10 units, which figure is added to the second column? Why? What order does tens of thousands belong in? How are numbers to be added, written? Which column is to be added first? How is addition proved? When the smaller of two numbers is given and also their difference, how is the larger number found. When dollars and cents are written, how many figures do the cents require? What is placed between the dollars and cents? On which side are the dollars written? What is the sign for United States money? What is one of ten equal parts of a unit called? What are two such parts called? Three such parts? What is one of two equal parts called? What is one of four equal parts of a number called? What is a half of a number? What is a third of anything? How many tenths in a unit? How many fifths; sixths; hundredths? How many cents in a dollar. If a dollar be thought of as a unit, what must a cent be called? How many dimes in a dollar? What part of a dollar is a dime? What part of a dime is a cent? What may we call a tenth of a tenth of anything? Is a cent a tenth of a tenth of a dollar? Repeat the table of U. S. money. What name may be given to any kind of parts of a thing? What kind of fractions of a dollar are dimes and cents? How many cents in 10 mills? How many in 20 mills; in 30 mills; in 70; in 100?

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SUBTRACTION.

Section V.

60. Subtraction *is the process of finding how much greater one number is than another.*

61. The excess is called the **Difference,** or **Remainder.**

62. The smaller number is said to be taken from the greater.

63. The greater is called the **Minuend,** because it has to be diminished; the smaller, the **Subtrahend,** because it has to be subtracted.

64. The **sign** of **subtraction** is a dash (—). It is called *minus,* and is written between the minuend and subtrahend. Thus 7—2=5 indicates that 7 *minus* 2 equals 5.

65. As *the difference added to the subtrahend equals the minuend,* the table of addition is, in fact, a table of subtraction.

Example.—A person had $465, and spent $253 of that sum. What remained?

$$\begin{aligned} 465 &= (400+60+5) \text{ dollars} \\ 253 &= (200+50+3) \text{ dollars} \\ \hline 212 &= (200+10+2) \text{ dollars} \end{aligned}$$

66. Axiom V. **One number is subtracted from another when the parts of the first are taken from those of the second.**

Example.—Subtract 176 from 452. *Ans.* 276.

$$\begin{aligned} 452 &= 300+14 \text{ tens}+12 \\ 176 &= 100+\ 7 \text{ tens}+\ 6 \\ \hline 276 &= 100+\ 70\ +\ 6 \end{aligned}$$

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67. RULE.—When any figure of the minuend is less than that of the same order in the subtrahend, take 1 from the next figure on the left and add it, *as* 10, to the figure to be diminished, and set down the difference between the subtrahend figure and this sum. In subtracting the next figure, the minuend figure must be reckoned as one less than it appears to be.

This method has of late been practised in school-work. It is based on Axiom IV.

68. *Method by Equal Additions.*

The ordinary method of Subtraction is based on the following truth:

69. AXIOM VI. **We do not alter the difference between two quantities if we add equal sums to both.**

Illustration.—(*a*) Suppose there are two heaps of stones, one of which contains 50 stones more than the other. Let 20 stones be added to each heap; still the excess in the larger is the same as before. We have altered the *size* of the heaps, but have not changed the difference.

Illustration.—(*b*) Find the difference between 8 and 10. 10—8=2.

If we add 2 to the minuend and add 2 to the subtrahend, we then have 12—10=2, as before.

Whenever any order of the subtrahend is greater than the corresponding order of the minuend, we may employ this method.

70. *Example.*—Subtract 17386 from 20245.

	Th.	H.	Tens.	Units.
20245+1000+100+10=20+		12+	14+	15
17386+1000+100+10=18+		4+	9+	6
2859	2	8	5	9

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We add 10 to the units of the minuend and 1 to the tens of the subtrahend; then add 10 to the tens of the minuend and 1 to the hundreds of the subtrahend; then add 10 to the hundreds of the minuend, and afterward 1 to the thousands of the subtrahend, the additions to each being equal.

Each of these methods has its advocates. That by equal additions was, until of late, in general use. Any person would probably prefer that to which he should first be accustomed.

71. *Rule for Subtraction.*

Write the less number under the greater, placing the figures as in addition.

Commence at the right hand and subtract each figure of the subtrahend from the one above it, setting the remainder under the figure subtracted.

If the figure above is the smaller, increase it by 10, *and then add* 1 *to the next lower figure before taking this next figure from the minuend.*

PROOF.—*Add the remainder and the subtrahend. The sum should equal the minuend.*

72. *Obs.*—In practice, consider the problem as a question in addition. Thus, in the problem

79234
32567
———

look at both numbers, and say, 7 *and* 7 (are 14), 7 *and* 6 (are 13), 6 *and* 6 (are 12), 3 *and* 6 (are 9), 3 *and* 4 (are 7). The numbers following the *ands* are to be written. The parentheses need not be spoken. In arithmetic, *Brevity, Rapidity,* and *Accuracy* are likely to be found together.

EXERCISE XIII.

1. From 1 ten take 7 units.
2. " 16 tens take 98 units.
3. " 40 hundreds take 125.

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4. From 111 thousand take 25 tens.
5. " 2 millions take 2 tens of thousands.
6. " 2 billions take two millions.
7. " 300003 take 30030.
8. Washington, born 1732; died 1799. How old?
9. Newton, " 1642; " 1727. "
10. Shakspere, " 1564; " 1616. "
11. Goldsmith, " 1728; " 1774. "
12. Gregory VII., " 1015; " 1085. "
13. Webster, " 1782; " 1852. "
14. W. Scott, " 1771; " 1832. "
15. Lingard, " 1771; " 1851. "
16. Thomas à Becket died 1170, aged 51. When born?
17. Joan of Arc " 1431, " 20. "

18–27. The following mountains are the highest in their respective ranges: Mt. St. Elias, 19,283 ft.; Mt. Uncompahgre (Rocky), 14,540; Mt. Whitney (S. Nev.), 14,887; Illampu, 24,812; Mt. Elbrooz, 18,526; Mt. Blanc, 15,784; Teneriffe, 12,182; Mt. Everest, 29,100; Mt. Ararat, 17,200; Mt. Washington, 6,288.

28. How far above Illampu is Everest?
29. " " Mt. Blanc "
30. " " Teneriffe "
31. How much higher is Mt. Blanc than Mt. Washington?
32. " " Whitney than Washington?
33. How much higher than Ararat is Illampu?
34. " " Washington is Mt. Blanc?
35. " " Washington is Whitney?
36. " " Mt. Marcy (5,467), is Washington?
37. What would be the height of Everest if it were capped by Mt. Elbrooz?
38. How high above Lake Titicaca (12,785) is Illampu?

Note.—Other questions for practice may be found in the chapter on Addition.

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39.	Take	$1.50	from	$3.00	44.	From	$80.00	take $1.80
40.	"	2.75	"	3.75	45.	"	8.00	" .10
41.	"	1.75	"	2.25	46.	"	10.00	" 10 mills
42.	"	4 cts.	"	4.00	47.	"	1.00	" 80 mills
43.	"	2.99	"	8.50	48.	"	20 dimes	" 20 cts.

73. Numbers written in a parenthesis are to be counted as one whole. Thus: (3+4)=7; (3+4)—2=5; 9—(2+6)=1; (18—5)—(6—2)=9.

49. 278—37=? 25+96+42+5—198=?

50. (1786—9—12)—(235+672—751)=? *Ans.* 1609.

51. 18+6+209+537—(628+31+19)=? *Ans.* 92.

The following problems may be used or omitted, at the option of the teacher.

74. TABLE OF DRY MEASURE.

2	pints	(pt.)	make	1	quart (qt.)
8	quarts		"	1	peck (pk.)
4	pecks		"	1	bushel (bu.)

Scale, 4, 8, 2

52. Add:

3	bu.	2	pks.	3	qts.	1	pt.
4	"	3	"	7	"	1	"
2	"	1	"	5	"	1	"
9	"	6	"	15	"	3	"
11	"	0	"	0	"	1	"

Explanation.—Adding each kind separately, there are 9 bu., 6 pks., 15 qts., and 3 pts. Now, 3 pts.= 1 qt.+1 pt. Set the 1 pt. down under the column of pts., and add the 1 qt. to 15 qts. There are then 16 qts., or 2 pks. and no qts. The 2 pks. being added to 6 pks., there are 8 pks., or 2 bushels and no pecks. These 2 bu. added to 9 bu. make 11 bu. As there are two denominations, or kinds of measures, between bushels and pecks, a cipher is written in each place as above, to prevent mistakes.

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(53) Add:

bu.	pk.	qt.	pt.
2	2	2	1
3	3	3	1
4	2	3	1
5	3	5	1
16	3	7	0

(54)

bu.	pk.	qt.	pt.
15	1	3	1
7	0	7	0
6	3	0	1
	2	2	1
29	3	5	1

75. TABLE OF LIQUID MEASURE.

4 gills	(gi.)	=1 pint	(pt.)
2 pints		=1 quart	(qt.)
4 quarts		=1 gallon	(gal.)

TABLE OF ENGLISH MONEY.

4 farthings	(far.)	= 1 penny	(*d.*)
12 pence		= 1 shilling	(*s.*)
20 shillings		= 1 pound	(£)
21 "		= 1 guinea	(gui.)

55. Take 1 bu. 1 pk. 1 qt. from 2 bu. 2 pks. 3 qts.

56. Take 2 bu. 4 pks. 5 qts. and 2 pts. from 3 bu. 3 pks. 3 qts. and 1 pt.

[57]

bu.	pk.	qt.	pt.
2	2	2	
1	1	1	1
1	1	0	1

[58]

	4	8	2	
3 bu.	3 pks.	3 qts.	1	pt.
2	4	5	2	
	2 pks.	5 qts.	1	pt.

It will be seen from [57] that 2 pts. were added to the minuend, as 1 pint was taken and 1 pint remained. 1 qt. (or 2 pints) must have been added to the subtrahend also, as naught remains of the 2 quarts in the minuend.

In [58] a qt., a pk., and a bush. were added successively to each compound number; first, a qt., in the form of 2 pts., to the right-hand part of the minuend; and then 1 qt. to the second part of the subtrahend, etc.

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76. *Questions on Subtraction.*

Define the following words, as used in Arithmetic: Subtraction, Difference, Remainder, Minuend, Subtrahend, Process. What is the sign of Subtraction? State a question in Subtraction, with its answer. Repeat Axiom IV. Repeat Axiom V. Repeat the Rule for Subtraction. How may the work of Subtraction be done most rapidly? How is the work proved? What three qualities should a pupil's work show? How are numbers written in a parenthesis to be thought of? What is the answer to this question: 8—(2+1) = what? How many pints in a quart; how many quarts in a peck; and how many pecks in a bushel? What letters stand for pints; for quarts; pecks; bushels? What abbreviations stand for farthing; for penny; shilling; pound? It is suggested that some of the abbreviations for the names of States should be called for; also that the spelling of the names of the States and of the leading cities should receive considerable attention.

Exercise XIV.

Add these first eight Powers of the nine digits (Art. 222).

1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256
3	9	27	81	243	729	2187	6561
4	16	64	256	1024	4096	16384	65536
5	25	125	625	3125	15625	78125	390625
6	36	216	1296	7776	46656	279936	1679616
7	49	343	2401	16807	117649	823543	5764801
8	64	512	4096	32768	262144	2097152	16777216
9	81	729	6561	59049	531441	4782969	43046721

Note.—These numbers may be referred to when the chapters on Powers and Progression are under discussion.

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UNITED STATES NAVY.

(Rep. Sec. of Navy, Dec., 1875.)

	Guns.	Tonnage.
1. *North Atlantic Station.*		
Worcester, flag-sh.	15	2,000
Powhatan,	17	2,182
Plymouth,	12	1,122
Ossipee,	8	828
Shawmut,	3	410
Kansas	3	410
Pinta,	2	306
2. *Iron-clads :*		
Dictator,	2	1,750
Canonicus,	2	550
Saugus,	2	550
Ajax,	2	550
Mahopac,	2	550
Manhattan,	2	550
Passaic	2	496
Catskill,	2	496
Lehigh,	2	496
Nahant,	2	496
Hospital Ship :		
Pawnee,	2	872
Total,	82	14,614
3. *South Atlantic Squadron.*		
Brooklyn, flag-sh.,	20	2,000
Monongahela,	11	960
Wasp,	1	365
Total,	32	3,325
Special Service.		
Minnesota,	45	3,000
European Station.		
Franklin, flag-ship,	39	3,173
Alaska,	12	1,122
Congress,	16	2,000
Juniata,	8	828
Total,	75	7,123
4. *North Pacific Station.*		
Pensacola, flag-sh.,	22	2,000
Lackawanna,	10	1,026
Portsmouth,	14	846
Tuscarora,	6	726
Total,	52	4,598
South Pacific Station.		
Richmond, flag-sh.,	14	2,000
Omaha,	12	1,122
Onward, store-sh.,	3	704
Total,	29	3,826
5. *Asiatic Squadron.*		
Tennessee, flag-sh.,	23	2,135
Kearsarge,	6	695
Yantic,	3	410
Ashuelot,	6	786
Saco,	3	410
Monocacy,	6	747
Palos,	6	306
Total,	53	5,489

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Iron-clad Vessels.

	Guns.	Tonnage.		Guns.	Tonnage.
6. Ajax,	2	550	7. Miantonomoh,	4	1,225
Amphitrite,	4	874	Monadnock	4	1,091
Canonicus,	2	550	Montauk,	2	496
Camanche,	2	496	Nahant,	2	496
Catskill,	2	406	Nantucket,	2	496
Colossus,	10	2,127	Oregon,	4	2,127
Dictator,	2	1,750	Passaic,	2	496
Hero,	1	483	Piscataqua,	1	483
Jason,	2	496	Puritan,	2	1,870
Lehigh,	2	496	Roanoke,	6	2,260
Mahopac,	2	550	Saugus,	2	550
Manhattan,	2	550	Terror,	4	1,085
Massachusetts,	4	2,127	Wyandotte,	2	550

How many guns?

" " tons?

Tonnage of the 9 largest?

8. STATE OF NEW YORK.

(32 Senators, 128 Assemblymen.)

Census. Year.	Whole Population.	Representative Population.	For a Senator.	For an Assemblyman.
1825,	1,614,458	1,530,648	47,864	11,966
1835,	2,174,517	2,042,599	63,828	15,957
1845,	2,604,495	2,399,548	74,992	18,748
1855,	3,466,212	2,797,510	87,420	21,855
1865,	3,827,818	3,393,698	106,052	26,513
1875,	4,705,208	4.376,458	136,736	34,180

SUBTRACTION.

Required the increase between 1825 and 1835=

" " " 1835 " 1845=

" " " 1845 " 1855=

" " " 1855 " 1865=

" " " 1865 " 1875=

" " " 1825 " 1845=

" " " 1855 " 1875=

" " " 1825 " 1855=

" " " 1845 " 1875=

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For problems in Multiplication, use 32 and 128 as multipliers of the foregoing quotas.

SIZE OF LEGISLATIVE BODIES.

9.

State.	Senate.	House.	Total.
Alabama,	33	100	133
Arkansas,	24	82	106
California,	40	80	120
Connecticut,	21	241	262
Delaware,	9	21	30
Florida,	24	53	77
Georgia,	44	175	219
Illinois,	25	90	115
Iowa,	40	100	140
Indiana,	50	98	148
Kansas,	25	75	100
Kentucky,	38	100	138
Louisiana,	36	101	137
Maine,	31	151	182
Maryland,	24	86	110
Massachusetts,	40	240	280
Michigan,	32	100	132
Minnesota,	22	47	69
Mississippi,	34	112	146
Missouri,	34	200	234

10.

State.	Senate.	House.	Total.
Nebraska,	13	39	52
Nevada,	19	38	57
N.Hampshire,	12	341	353
New Jersey,	21	60	81
New York,	32	128	160
N. Carolina,	50	120	170
Ohio,	36	105	141
Oregon,	16	34	50
Pennsylvania,	33	100	133
Rhode Island,	36	72	108
S. Carolina,	33	124	157
Tennessee,	35	75	110
Texas,	30	60	90
Vermont,	30	241	271
Virginia,	43	132	175
W. Virginia,	24	65	89
Wisconsin,	33	100	133
	1,112	4,086	5,208

Note.—The above comprises the States of the Union December, 1875. For oral problems in Subtraction, give the *totals* as minuends and the numbers in the *Senate* column as subtrahends; the second, or Assembly column, will furnish the differences.

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EXERCISE XV.

Problems in Addition and Subtraction.

AREAS AND POPULATION OF NEW ENGLAND AND MIDDLE STATES (1870).

	Area.	Pop.		Area.	Pop.
Me.,	35,000	626,915	N. J.,	8,320	906,096
N. H.,	9,300	318,300	Penn.,	46,000	3,521,951
Vt.,	10,200	330,551	Md.,	11,124	780,894
Mass.,	7,800	1,457,351	Del.,	2,120	125,015
R. I.,	1,306	217,353	Va.,	38,348	1,225,163
Conn.,	4,700	537,454	W. Va.,	23,000	442,014
N. Y.,	47,000	4,382,759			

1, 2. Find the totals of each group. (2 answers.)

3. How many more people has New York than New Eng.?

4. How many more sq. miles has New Eng. than N. Y.?

5. How many more people has R. I. than Del.?

6. How much larger than R. I. is Del.?

7, 8. Compare the areas and pop. of Vt. and N. H. (2 ans.)

9–12. Compare Mass. with N. J.; Penn. with N. Y. (4 ans.)

13–16. Va. with W. Va.; Conn. with Md. (4 ans.)

17. The population of Portland, Me., is 31,413; of Manchester, N. H., 23,536; of Burlington, Vt., 14,387; of Boston, 250,526; of Providence and N. Providence, R. I., 89,399; of New Haven, Conn., 50,480; of New York, 942,292; of Newark, N. J., 105,059; of Philadelphia, 674,022; of Wilmington, Del., 30,841; of Baltimore, 267,354; of Richmond, Va., 51,038; of Wheeling, W. Va., 19,280. Required their total population (1870).

18. Compare the united population of the cities having Bs for initials with the total population of those beginning with P.

19. Compare the sum of the Ns with the sum of the Ps.

20. Compare the united population of all the cities named with what remains when this number is taken from the sum of the population of the 13 States already named.

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NORTH CENTRAL STATES AND CHIEF CITIES.

	Area.	Pop.		Pop.
Ky.,	37,680	1,321,000	Louisville,	100,753
O.,	39,694	2,665,000	Cincinnati,	276,239
Ind.,	33,800	1,680,000	Indianapolis,	48,244
Mich.,	56,451	1,184,000	Detroit,	101,000
Ill.,	55,410	2,539,000	Chicago,	298,977
Wis.,	53,924	1,054,000	Milwaukee,	71,440
Minn.,	83,531	439,000	St. Paul,	20,030
Ia.,	55,045	1,194,000	Davenport,	20,038
Mo.,	65,350	1,721,000	St. Louis,	310,864

21, 22. How much larger is Minn. than O.? Ill. than Ind.?

23, 24. Ill. than Ia.? O. than Ind.?

25. How much larger is Minn. than both O. and Ky.?

26. Required the difference in pop. between St. Louis and Cincinnati.

27. Between Chicago and Cincinnati?

28. Between Milwaukee and Louisville?

29. How much greater in extent are these 10 States than the first 13 mentioned?

EXTENT AND POPULATION OF SOUTHERN AND WESTERN STATES.

	Area.	Pop.		Area.	Pop.
N. C.,	50,704	1,071,400	Ark.,	52,198	484,500
S. C.,	34,000	705,000	Tex.,	274,356	818,000
Ga.,	58,000	1,884,000	Kan.,	81,318	364,000
Fla.,	59,300	188,000	Neb.,	75,995	122,000
Ala.,	50,722	997,000	Cal.,	188,981	560,000
Miss.,	47,156	828,000	Or.,	95,274	90,000
Tenn.,	45,600	1,258,000	Nev.,	104,125	42,000
La.,	41,346	727,000			

30. What is the extent of the first 9?

31. What is the extent of the last 5?

32. How much larger is Tex. than Cal.?

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33. How much larger is Tex. than New Eng.?

34. How much larger is Kan. than New Eng.?

35. How much larger is Tex. than New England and the Middle States taken together?

France contains 204,000 sq. miles; Germany, 208,000; Italy, 114,000; Holland, 12,700; and Belgium, 11,300.

36. How much larger than Md. is Holland?

37–39. Belgium than N. J.? Italy than Or.? Tex. than France?

40, 41. Germany than France? The Southern States than France and Germany if taken together?

42. The population of the following **European cities** is expressed in thousands, viz.: London, 3,250; Paris, 2,000; Berlin, 900; Vienna, 850; St. Petersburg, 700; Stockholm, 135; Amsterdam, 270; Brussels, 175; Copenhagen, 180; Geneva, 70; Madrid, 350; Lisbon, 224; Rome, 200; Athens, 50; Constantinople, 1,000.

Arrange the names in their order as respects population, with the corresponding numbers on their right, and letter the phrases *a*, *b*, *c*, etc.; then find the difference (in thousands) between (1) *a—b*; (2) *b—c*; (3) *d—e*; (4) *e—f*; etc. Thus: (1) 3250—2000=1250; (2) 2000—1000=1000.

From their geographies scholars may make similar tables referring to Asia, S. America, U. S., etc.

Exercise XVI.

1. A boatman set out to row the length of the Erie Canal, 364 miles; after rowing 16 miles the first day, 22 the second, 24 the third, and 22 miles for each of the six following days, how far was he from the end of his journey?

2. I deposit in bank $1,240. I first check out $375 and afterward $422; what still stands to my credit?

3. Eugene puts into a savings-bank $1.12, then .46, then $2.24, then $1.92, then $4.96; he wishes to draw out $10; what would remain?

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4. If in one pile of bricks there are 7896 and in another 3389; how many are there in the one more than in the other?

5. In building a house there were taken 51260 bricks from a pile of 71964; how many remained?

6. A man purchased at various times 11400+1326+4281+100000+7962 bricks. He sold 4162+114222 of these; how many remained?

7. A farmer owned two farms, one of which contained 724 acres, the other 926 acres. He gave to his son Henry 216 acres, to John 119, to Sarah 112, to Jane 225, and kept the rest in the homestead; how many acres did he retain?

8. How many years between the settlement of Virginia, 1607, and the Revolutionary War, 1775?

9. How many years between the birth of Washington, 1732, and the outbreak of the War of Secession, 1861?

10. How long after the war with Mexico, 1846, to the admission of Nevada, the 36th State, 1864?

11. Lawrence received 15 cents from his father, 12 from his mother, 10 from each of his three aunts, and 24 from his uncle Thomas. He spent 14 cents for marbles, 5 for a top, 10 for fire-crackers, gave 5 to a beggar, then bought a knife, a hoop, and a kite, giving for each 8 cents; how much money had he left?

12. A man borrows 7 ten-dollar bills and 3 one-dollar bills, and pays at one time 4 ten-dollar bills and 5 one-dollar bills; how many ten-dollar bills and one-dollar bills must he afterward pay to cancel the debt?

Ans. 2 ten-doll. bills and 8 one-doll.

13. The greater of two numbers is 24, and the less is 16; what is their difference?

14. The greater of two numbers is 24, and their difference 8; what is the less number?

15. The sum of two numbers is 40, the less is 16; what is the greater?

16. A tree 68 feet high was broken off by the wind; the top, which fell, was 49 feet long; how high was the stump?

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17. How many years from the invention of movable types, 1452, to the year of the invention of the telegraph, 1832?

18. A man carried his produce to market: he sold his pork for 45 dollars, his cheese for 38 dollars, and his butter for 29 dollars; he received in pay salt to the value of 17 dollars, 10 dollars' worth of sugar, 5 dollars' worth of molasses, and the rest in money; how much money did he receive?

Ans. 80 dollars.

19. A boy bought a sled for 28 cents, and gave 14 cents to have it repaired; he sold it for 40 cents; did he gain or lose by the bargain, and how much?

20. One man travels 67 miles in a day, another man follows at the rate of 42 miles a day; if they both start from the same place at the same time, how far will they be apart at the close of the first day? of the second? of the third? of the fourth?

21. One man starts from Boston Monday morning, and travels at the rate of 40 miles a day; another starts from the same place Tuesday morning, and follows on at the rate of 70 miles a day; how far are they apart Tuesday night?

Ans. 10 miles.

22. A man owing 379 dollars paid at one time 47 dollars, at another time 84 dollars, at another time 23 dollars, and at another time 143 dollars; how much did he then owe?

Ans. 82 dollars.

23. A man has property to the amount of 34,764 dollars, but there are demands against him to the amount of 14,297 dollars; how many dollars will be left after the payment of his debts?

24. Four men bought a lot of land for 482 dollars; the first man paid 274 dollars, the second man 194 dollars less than the first, and the third man 20 dollars less than the second; how much did the second, the third, and the fourth man pay?

Ans. { The second paid 80.
The third paid 60.
The fourth paid 68.

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25. A man, having 10,000 dollars, gave away 9 dollars; how many had he left? *Ans.* 9991 dollars.

26. John has 240 sheep more than Joseph, and 125 less than James, who has 485; and Jerry has as many as John and Joseph together, lacking 20 sheep. How many sheep have John, Joseph, and Jerry respectively; viz., how many has each?

MULTIPLICATION.

Section VI.

77. ***Multiplication*** is the process of repeating one number as many times as there are units in another.

78. The number repeated is called the **Multiplicand.**

79. The number which shows how many times the multiplicand is to be repeated, or added, is called the **Multiplier.**

80. The result is called the **Product.** The multiplier and multiplicand are called **Factors** of the *Product.*

81. The **sign of multiplication** is an oblique cross, $\times$. It is placed between the factors.

82. A problem in multiplication is written thus: $2\times5=10$; and is read thus: 2 multiplied by 5 equals 10.

83. When both factors are abstract, either may be taken as the multiplier. The multiplier must be **abstract.** The product is like the multiplicand; thus: $4\times6=24$; 6 cents $\times$ $4=24$ cents; 7 men $\times$ $3=21$ men.

84. Multiplication is a **short method of adding** several **equal** numbers.

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85. MULTIPLICATION TABLE.

2 times	3 times	4 times	5 times	6 times	7 times
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7
2 " 4	2 " 6	2 " 8	2 " 10	2 " 12	2 " 14
3 " 6	3 " 9	3 " 12	3 " 15	3 " 18	3 " 21
4 " 8	4 " 12	4 " 16	4 " 20	4 " 24	4 " 28
5 " 10	5 " 15	5 " 20	5 " 25	5 " 30	5 " 35
6 " 12	6 " 18	6 " 24	6 " 30	6 " 36	6 " 42
7 " 14	7 " 21	7 " 28	7 " 35	7 " 42	7 " 49
8 " 16	8 " 24	8 " 32	8 " 40	8 " 48	8 " 56
9 " 18	9 " 27	9 " 36	9 " 45	9 " 54	9 " 63
10 " 20	10 " 30	10 " 40	10 " 50	10 " 60	10 " 70
11 " 22	11 " 33	11 " 44	11 " 55	11 " 66	11 " 77
12 " 24	12 " 36	12 " 48	12 " 60	12 " 72	12 " 84

8 times	9 times	10 times	11 times	12 times
1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 " 16	2 " 18	2 " 20	2 " 22	2 " 24
3 " 24	3 " 27	3 " 30	3 " 33	3 " 36
4 " 32	4 " 36	4 " 40	4 " 44	4 " 48
5 " 40	5 " 45	5 " 50	5 " 55	5 " 60
6 " 48	6 " 54	6 " 60	6 " 66	6 " 72
7 " 56	7 " 63	7 " 70	7 " 77	7 " 84
8 " 64	8 " 72	8 " 80	8 " 88	8 " 96
9 " 72	9 " 81	9 " 90	9 " 99	9 " 108
10 " 80	10 " 90	10 " 100	10 " 110	10 " 120
11 " 88	11 " 99	11 " 110	11 " 121	11 " 132
12 " 96	12 " 108	12 " 120	12 " 132	12 " 144

86. In learning the table, remember that either factor may be used or considered as the multiplier and the result will be the same. Thus 3 times 2 are 6, and 2 times 3 are 6; $5 \times 12 = 12 \times 5$, etc. Instead of saying two times one, or two times five, we generally say twice two, or twice five, etc. Use an irregular table for *blackboard* drill.

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EXERCISE XVII.

ORAL EXERCISE.

87.	10	20	30	40	50	60	70	80	90
	9	18	27	36	45	54	63	72	81
	8	16	24	32	40	48	56	64	72
	7	14	21	28	35	42	49	56	63
	6	12	18	24	30	36	42	48	54
	5	10	15	20	25	30	35	40	45
	4	8	12	16	20	24	28	32	36
	3	6	9	12	15	18	21	24	27
	2	4	6	8	10	12	14	16	18
	1	2	3	4	5	6	7	8	9

Count upward by 2s, by 5s, by 8s, etc., etc.

At 3 cents apiece, what will 4 pears cost?

What is the sum of 6 threes? 5 fours?

Required, cost of 5 pencils, at 5 cents each?

At 9 cents each, what will 7 primers cost?

How far can a man walk in 4 hours, if he travels 4 miles an hour? How far in 11 hours?

How many days in 6 weeks?

How many quarts in 3 pecks?

---

1. What cost 2 apples, at 2 cents apiece?

Note.—Since one apple cost 2 cents, 2 apples will cost 2 times 2 cents, which are 4 cents.

2. What cost 2 lemons, at 3 cents apiece?

3. What cost 2 oranges, at 4 cents apiece?

4. What cost 2 oranges, at 5 cents apiece?

5. What cost 2 pounds of rice, at 10 cents a pound?

6. Bought 2 writing-books, at 7 cents apiece; what did they cost?

7. If one pound of sugar cost 8 cents, what will 2 pounds cost?

8. If one pound of veal cost 9 cents, what will 2 pounds cost?

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9. At 10 cents each, what will 2 spelling-books cost?

10. What cost 2 quarts of berries, at 11 cents a quart?

11. What cost 2 pine-apples, at 12 cents apiece?

12. What cost 3 pears, at 4 cents apiece?

13. What cost 3 quarts of milk, at 9 cents a quart?

14. What cost 3 yards of braid, at 6 cents a yard?

15. If a horse will trot 7 miles in one hour, how far will he trot in 3 hours?

16. If one pound of raisins cost 8 cents, what will 3 pounds cost?

17. Bought 3 yards of cloth at 9 cents a yard; how much did it cost?

18. At 3 dollars a pair, what will ten pairs of boots cost?

19. At 3 dollars apiece, what will 11 hats cost?

20. What cost 3 loaves of bread, at 12 cents a loaf?

21. What cost 4 bushels of cranberries, at 3 dollars a bushel?

22. At 4 dollars a yard, what cost 4 yards of broadcloth?

23. At 4 cents a pound, what cost 6 pounds of rice?

24. What cost 5 vests, at 4 dollars apiece?

25. What cost 4 cords of wood, at 7 dollars a cord?

26. What cost 10 skeins of silk, at 4 cents a skein?

27. At 4 cents apiece, what cost 8 oranges? 9 oranges? 11 oranges? 12 oranges?

28. How many are 5 times 5? 5 times 6? 5 times 8? 5 times 9?

29. At 5 cents apiece, what cost 7 lead-pencils? 10 lead-pencils? 11 lead-pencils? 12 lead-pencils?

30. When 6 dollars are paid for a cord of wood, what must be paid for 5 cords? For 6 cords? For 7 cords?

31. How many are 6 times 8? 6 times 9? 6 times 10? 6 times 11? 6 times 12?

32. If a horse travel 7 miles in 1 hour, how far will he travel in 7 hours? In 9 hours? In 11 hours?

33. At 7 cents a pound, what will 8 pounds of beef cost? 10 pounds? 6 pounds? 12 pounds?

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34. When flour is 8 dollars a barrel, how much must be paid for 5 barrels? 9 barrels? 8 barrels? 10 barrels? 11 barrels? 12 barrels?

35. At 9 cents a pound, what cost 6 pounds of coffee? 9 pounds? 11 pounds? 10 pounds? 12 pounds?

36. What cost 10 yards of cambric, at 12 cents a yard?

37. If a quantity of provisions will supply 11 men 8 days, how long will it supply one man?

38. William sold ten doves at 10 cents apiece. How much did he receive for them?

39. What will 11 dozen of eggs cost, at 10 cents a dozen? At 11 cents? At 12 cents?

40. If 12 men can do a piece of work in 12 days, how long will it take one man?

When either factor is larger than 12, we may write the multiplier under the multiplicand, and proceed as follows:

1. 9423
4
37692

Ex. 1 analyzed:

9	4	2	3
			4
36	16	8	12
37	6	9	2

The tens figure of 12 may be written with 8 tens, making 9 tens; the tens figure of the 16 hundreds, written with 36 thousands, would make 37 thousands. Thus, it appears that the sum of the separate products will equal 37692.

88. When the Multiplier is 12, or less.

Rule.—*Write the multiplier under the multiplicand; find the product of the multiplier and the units figure above it; set down the right-hand figure of this product, and, reserving the left-hand figure, add it to the product of the multiplier into the next figure above; set down the right-hand figure of this second result in the tens' place, and add the left-hand*

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figure to the next product; and so proceed until every figure of the multiplicand has been multiplied.

EXERCISE XVIII.

1–5.	Multiplicand,	57436	5432	2345	9054	152634
	Multiplier,	2	3	4	5	6
	Product,	114872				
6–10.	Multiplicand,	17048	81034	5419	41261	4230
	Multiplier,	7	8	9	11	12
	Product,					

11. If a man travel 48 miles a day, how many miles will he travel in 9 days? *Ans.* 432 miles.

12. What is the product of 956 multiplied by 12? *Ans.* 11,472.

13. What will 325 barrels of flour come to at 7 dollars a barrel? *Ans.* $2,275.

14. 320 rods make a mile. How many rods are there in 8 miles? *Ans.* 2560.

89. MULTIPLICATION OF DOLLARS AND CENTS.

15. $75.69 = Multiplicand.
7 = Multiplier.

$529.83 = Product.

Explanation.—Beginning at the right hand, or lowest denomination of the multiplicand, we have 7 times 9 hundredths = 63 hundredths, or 6 tenths and 3 hundredths.

Writing the 3 in the hundredths' place, and reserving the 6 tenths to add to the product of the tenths by 7, we have 7 times 6 tenths = 42 tenths and 6 tenths added, = 48 tenths, = 4 units and 8 tenths.

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Writing the 8 tenths, and reserving the 4 units to add to the product of the units by 7, we have 7 times 5 units=35 units, and 4 units added=39 units=3 tens and 9 units.

Writing the 9 units, and reserving the 3 tens to add to the product of the tens by 7, we have 7 times 7 tens=49 tens, and 3 tens added=52 tens, which, being our last product, we write. The result, then, is $529.83.

Note.—As soon as practicable, the explanation should be abbreviated so as to name only results. Thus: 63 hundredths; 42, 48 tenths; 35, 39 units; 49, 52 tens? *Ans.* $529.83.

16. If 1 horse cost $92.50, what will 4 horses cost?

17. If 1 cow cost $45.75, what will 5 cows cost?

18. If 1 wheelbarrow is worth $8.63, what is the value of 6 wheelbarrows?

19. John bought 9 pigeons at 48 cents apiece, and James bought 7 times as many at the same price; what did James pay in all?*

20.* If one pint of peanuts cost 15 cents, what will 7253 quarts cost?

21, 22.* What will 15675 pecks cost? 3564 bushels?

23.* If 1 gill of molasses is worth 3 cents, what will 16 gallons cost?

24. There are 12 pence in 1 shilling; how many pence in 42654 shillings?

25.* How many quarts in 192468 pecks?

26.* How many gills in 8263 gallons?

27–30. $84687 \times 3 = ?$ $2149 \times 6 = ?$ $7428 \times 7 = ?$ $416 \times 9 = ?$

31–34. $\$200.76 \times 9 = ?$ $\$41.68 \times 3 = ?$ $\$3412.99 \times 2 = ?$ $4168 \times 12 = ?$

35. What will 4 hogsheads of cider cost at $9.25 each?

36. Req. the prod. of 111,946,492 by 6.

Ans. 671,678,952.

* See Tables in chapter on Subtraction.

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37. Req. the prod. of 13,698,246×12. *Ans.* 164,378,952.
38. Req. the prod. of 33,081,425×11. *Ans.* 363,895,675.
39. Req. cost of 564 hhds. of molasses at $36 per hhd.

Solution.—At $6 per hhd. the cost would equal $6×564, or $3384; and at $36, the cost will equal $3384×6, or $20304.

90. A composite number is one that can be made by multiplying two or more numbers together.

To Multiply by a Composite Number.

Rule.—*First multiply the given quantity by one factor, then multiply the result by the next factor, and continue until all have been employed as multipliers.*

Exercise XIX.

1–3. Multiply 87,412 by 35; 847,209 by 63; 8291 by 45.
4–6. 20974 by 144; 962187 by 24; 23972 by 28.
7. How many sheets of paper in 359 reams, there being 24 sheets in a quire and 20 quires in a ream? 359×4×5×6×4=? *Ans.* 172,320.
8, 9. How many sheets in 186 reams? In 242 reams?
10. If a man walk up and down a flight of 27 steps 84 times, how many steps does he take upon it?
11. If a hhd. hold 63 gallons, each containing 4 quarts, each equal to 2 pints, each measuring 4 gills, how many gills in 128 hhds?

91. As the value expressed by a figure depends on the order in which it stands, *each removal one place toward the left multiplies its value by* 10. A naught written at the right of a line of figures, locates each one of the figures in the next higher order; hence, **annexing a naught multiplies the whole number by 10.** The annexing of two naughts to a quantity, makes it 100 times as large as before. Thus 12 equals 12 units, 1200 equals 12 hundreds.

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92. WHEN CIPHERS STAND AT THE RIGHT OF EITHER OR BOTH FACTORS.

RULE.—*Multiply the significant figures together, and annex as many ciphers to the product as there are on the right of the given factors.*

1. $126{,}000 \times 9{,}000 = 1{,}134{,}000{,}000$.

2. $48{,}296{,}300 \times 84{,}000 = 4{,}056{,}889{,}200{,}000$; for, $84 = 7 \times 12$; $482{,}963 \times 7 = 3{,}380{,}741$; $3{,}380{,}741 \times 12 = 40{,}568{,}892$.

3. Multiply 496,142 by 54,000.

4. 341,620 by 4,800.

Example 1.—Suppose it were required to multiply 27 by 32. This is as if it were desired to multiply 27 by 30 *and* by 2. Now, $27 \times 30 = 810$ and $27 \times 2 = 54$. Adding both products and there results 864. The work might be set down thus:

27	*Ex.* 2. $424 \times 569 = 241{,}256$
32	
	$424 \times 9 = 3{,}816$
54	$424 \times 60 = 25{,}440$
810	$424 \times 500 = 212{,}000$
864	Sum of these products $= 241{,}256$

If each child in a school of 135 pupils had 5 marbles, and each in a school of 73 had 8, how many would there be in all?

In five chests there are 2000 oranges; one contains 32 dozen, and 3 others have in each 35 dozen and six; how many are in the fifth?

If a boy read 28 pages every day except Monday, counting 4 weeks to the month, and 12 months to the year; how many pages less than 10,000 will he have read in a year?

What is the difference between six dozen dozen and a half a dozen dozen?

What does $(24 \times 12 \times 12) - (22 \text{ doz.} + 42 \text{ doz.} + 32) = ?$

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93. Axiom VII. **We multiply a number by another, when we multiply every part of the one by every part of the other and add the results.**

Ex. 4.

	4	6	3	5
			1	7
	28	42	21	35
	40	60	30	50
	68	102	51	85
7	8	7	9	5

Analysis.—$7\times5=35$; 7×3 tens$=21$ tens; 7×6 hundreds$=42$ hundreds; 7×4 thousands$=28$ thousands. 1, as it stands in the *second order*, is really 10; $10\times5=50$; 10×3 tens $=30$ tens; 10×6 hundreds$=60$ hundreds; 10×4 thousands$=40$ thousands. As the units have been classed with the units, and the tens with the tens, we may add them as they stand, and have 68 thousands$+$102 hundreds$+$51 tens$+$85 units. Now, by Axiom III., the sum of these may be written thus: 78795, as above.

Note.—It may be well to exercise a class on the preceding analysis until they can analyze other operations readily and thoroughly. Use vertical lines for a time.

4635
17

32445
4635

78795

94. When the Multiplier is more than Twelve.

Rule.—*Write the multiplier under the multiplicand, placing units under units, tens under tens, etc. Multiply by each digit separately, setting the first figure of each product directly under its multiplier. Add the partial products as they stand to find the total product.*

Example 5.—If one tree bear 16784 apples, how many apples will 2006 trees yield, if all bear equally? *Ans.* 33,668,704.

16784
2006

100704
33568

33668704

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Exercise XX.

1–4. Multiply 213459 by 46; by 73; by 58; by 175.

5, 6. Multiply 3456 by 789 and by 567.

7. What does $2,703,682 \times 8,409 = ?$ *Ans.* 22,735,261,938.

8. What is the product of 15,569,800 by 8,300?
Ans. 129,229,340,000.

9. $35,860,205 \times 365 = ?$ *Ans.* 13,088,974,825.

10. There are 5,280 feet in a mile; how many inches?

11. The girth of the earth at the Equator is 24,899 miles; how many inches does it measure?

12–19. The Mississippi River is 4,200 miles long; the Nile, 4,000; the Amazon, 3,750; the Yenisei, 3,400; the Volga, 2,400; the Murray, 1,500; the Hudson, 350; the Thames, 140. Length in feet of each? (Where are these rivers?)

20. How many square yards in a room 42 ft. by 15 ft.?

Obs.—Multiply the length in yards by the width in yards —*i. e.*, $14 \times 5 = 70$.

21. How many square feet of boards in a platform 164 ft. by 122? In one 48 ft. by 28? (Multiply length by width.)

22. The United States (excluding Alaska) is about 2200 miles from E. to W., and 1420 miles from N. to S. Required, its area. *Ans.* 3,124,000.

23. How many mills in 1913 dollars?

24. How many cents in 2564 dollars?

25. 2000 lbs. make one ton; 16 ozs. make 1 lb. How many ounces in 4 tons? How many in 12 tons? In 16?

26. How many pence in £25 English money? In £44?

27. How many gills in 12 gallons? In 49 gallons? In 200?

95. *Review.*

Exercise XXI.

1. Multiply the sum of 42 and 16 by $3 \times 19 \times 24 \times 8$.

2, 3. $(9 \times 3) + (15 \times 5) = ?$ $(6 \times 64 \times 14) + (4 \times 42 \times 8) = ?$

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4. 3 packages of sugar, each weighing 7 lbs., at 13 cts. a lb., and 4 hams, each weighing 17 lbs., at 19 cts. a lb., are worth how many dollars and cents?

NOTE.—Set off with a period two figures on the right for cents; the figures on the left of the point will stand for dollars.

5. 7 hhds. of sugar cost $47 each, and were sold at $63 each. What was gained?

6. A planter sold 1546 lbs. of cotton at 11 cents; after receiving 540 lbs. of bacon at 7 cents, and 212 gallons of molasses at 26 cents, in exchange, how much cash was due?

Ans. $77.14.

7. 84 oranges were divided between Tom, Dick, and Harry. Tom received 24; Dick, 3 times as many as Tom, lacking 43; and Harry was satisfied. Why?

8. From each of 12 logs were cut 22 boards, averaging 15 ft. in length by 2 ft. in width. This lumber was sold at 13 cents a square foot. The buyer gave a five-hundred-dollar note in payment. What change did he receive?

9. There is a certain chestnut-tree whose trunk divides into 4 great limbs. Each limb has 7 arms; each arm supports 9 branches; each branch sends out 11 boughs; and from each bough there spring 12 sprays; from each spray jut 13 twigs; every twig bears 15 chestnuts. How many chestnuts will fall from that tree?

10. Explain the following: $954,682 \times 7515 = 7,174,435,230$.

10.

9546820	(10).
4773410	(5).
14320230	
71601150	($5 \times 15 \times 100$).
7174435230	(7515).

96. *Questions to follow Multiplication.*

Count 10. Count 20 by 2s; 30 by 3s; 50 by 5s; 80 by 8s. Define these arithmetical terms: Multiplication, Multiplicand, Multiplier, Product. What terms must be used in

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questions that refer to Subtraction? What is the sign of Multiplication? State a question in Multiplication together with its answer. Tell which the Product is, and which the Multiplicand? What are Factors? What kind of number is the Multiplier? What kind of units will those of the product be like? State the rule for the multiplication of large numbers by 12 or less. How may a number be increased a hundred times; a thousand times; ten thousand times? How may multiplication by a composite number be done? Give an example of a composite number, and name its factors Repeat Axiom VI.; Axiom VII. Repeat the rule for multiplying by a large number.

DIVISION.

Section VI.

How many 2s in 4? How many in 6? In 10? In 14?
How many 3s in 9? How many in 15? In 21?
If 8 be divided into 4 equal parts, what is one part?
What number multiplied by 7 will produce 14?
8 times what number makes 24?

97. ***Division teaches,***

I. **The process of finding the number of times that one number is contained in another.**

II. **The process of dividing a number into equal parts.**

III. **The process of finding a factor when a product and the other factor are given.**

98. The **Dividend** is the number to be divided.

99. The **Divisor** is the dividing number.

100. The **Quotient** shows how many times the divisor is

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contained in the dividend. *Quotiens* (Latin) means "How often?"

The number left over, if any there be, is called the **Remainder.**

101. The **Sign of division** is a short horizontal line between two dots. Thus, $12 \div 3$ is read: 12 divided by 3. 12 is the dividend, 3 is the divisor, and 4 is the quotient.

102. The dividend should be written *before* the sign, and the divisor after it.

103. Division is often shown by writing a divisor under a dividend, with a line between. Thus, $\frac{14}{2}=7$ is read 14 divided by 2 equals 7.

104. A problem in division may be stated in various forms. The division of 21 by 7 is required by each of the following:

a. Divide 21 by 7.

b. How many times is 7 contained in 21?

c. What number multiplied by 7 will produce 21?

d. What is one of the 7 equal parts of 21?

e. If 7 is one factor of 21, what is the other?

f. Divide 21 into as many parts as there are units in 7.

g. How many times can 7 be subtracted from 21?

NOTE.—Multiplication is the reverse of Division. (*See c and e.*) Division is but a short method of Subtraction. (*Vide g.*) To divide 21 by 7 is but to take 7 three times from 21; thus: 21—7—7—7=14, 7, 0. In training pupils to use the multiplication-table, for purposes of division, at first, formulate the questions thus: 4 times 5 are what? 3 times 7 are how many? Then alternate with such questions as these: 4 times what are 20? 3 times what are 21?

(1) Divide 225 by 5.

SOLUTION 1.		SOLUTION 2.
5)200+20+5 or	5)225	6)756
40+ 4+1=45	45	126

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105. EXPLANATION OF SOLUTION 2.—Writing the divisor on the left of the dividend, and drawing a line under the latter, we first find the quotient arising from dividing the hundreds. The quotient, 1, is one hundred, and is written under the hundreds of the dividend. The hundred left undivided, together with the 5 tens, make 15 tens, which give a quotient of 2 tens; write this under the tens. 6 times 2 tens are but 12 tens, so 3 tens remain to be considered; these, with the units, equal 36 units, which contain 6, six times. We might analyze the work thus:

$$756 = 600 + \overbrace{100 + 20} + \overbrace{30 + 6}.$$

In each partial division, we take so much of the dividend as will exactly contain the divisor, and add the remainder to the quantity on the right; then take so much of this sum as will again exactly contain the divisor; and so continue till nothing remains. If there be a remainder, the divisor is to be written under it, and the expression is to be annexed to the quotient.

106. WHEN THE DIVISOR IS LESS THAN 10.

RULE.—*Place the divisor at the left of the dividend with a line of separation. Find how many times it is contained in the first figure or figures of the dividend, and set the result under the figure divided. If there be no remainder, divide the next figure on the right and set the quotient under it. If there be a remainder, prefix it to the next figure of the dividend and divide as before. Whenever the divisor is not contained, write a cipher in the quotient, and, prefixing the partial dividend to the next dividend figure, proceed as before.*

PROOF.—*Multiply the quotient by the divisor, and add in the remainder, if any; the result should equal the dividend.*

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8)4963512

600000 The rem. 100000+60000 is the second dividend.

20000

0000

400 The rem. 300+10 is the fifth dividend.

30

9 Usual process: 8)4963512

620439

620439

Exercise XXII.

Find the 42 quotients required by the following:

Divide each of these 6 dividends, viz., 116942, 700415, 30,405,060, 7194200, 842900, 7500004, by each of these 7 divisors, viz., 4, 5, 6, 7, 9, 11, and 12.

107. Teachers should not fail to induce their pupils to consider the *close correspondence of Division and Multiplication.*

In every case of Division, the Divisor and Quotient are *Factors* of the Dividend.

108. Divisor × quotient = dividend; multiplicand × multiplier = product. Hence the principles, or Axioms, of Multiplication and Division are practically the same.

I. **Multiplying the divisor, or dividing the dividend, divides the quotient.**

II. **Multiplying the dividend, or dividing the divisor, multiplies the quotient.**

III. **Multiplying both dividend and divisor leaves the quotient unaltered.**

IV. **Dividing both dividend and divisor will not alter the quotient.**

I. illustrated: $\frac{24}{6}=4$; $\frac{24}{6\times 2}=2$; $4\div 2=2$; so also, $\frac{24\div 2}{6}=2$

II. illustrated: $\frac{24}{6}=4$; $\frac{24\times 2}{6}=8$; $4\times 2=8$; so also, $\frac{24}{6\div 2}=8$

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III. illustrated: $\frac{24}{6}=4$; $\frac{24\times3}{6\times3}=\frac{72}{18}=4$

IV. illustrated: $\frac{24}{6}=4$; $\frac{24\div3}{6\div3}=\frac{8}{2}=4$

EXERCISE XXIII.

What other dividend and divisor will give a quotient equal to—

$\frac{120}{12}$	$\frac{29}{7}$	$\frac{37}{5}$	$\frac{15}{3}$	$\frac{496}{20}$	$\frac{47}{3}$	$\frac{91}{13}$
$\frac{84}{6}$	$\frac{132}{18}$	$\frac{524}{8}$	$\frac{37}{11}$	$\frac{13}{5}$	$\frac{19}{17}$	$\frac{69}{8}$

109. ORAL LESSON.

What is one-half of one-half of 8? How much is $\frac{1}{3}$ of 12 divided by 2?

What would be one-sixth part of the quotient of 9 into 108? What would $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{3}$ of 144 be?

What shall we call the one-fourth part of one-sixth of 48?

$$\begin{array}{r|l} 6)48 & \textit{Analysis.}\text{—One-sixth of 48 is 8;} \\ 4)\overline{8} & \text{one-fourth of 8 is 2.} \\ \overline{2} & \end{array}$$

$$\begin{array}{r|l} 4\times6=24 & \text{We may divide by 24} \\ 24)48 & \text{by dividing by 6 and} \\ \overline{2} & \text{then by 4. } 48\div24=2. \end{array}$$

110. AXIOM VIII. **We divide by a number when we divide by its factors successively in such a mode that the quotient of the first shall be divided by the next factor, and its quotient by the next, and so on.**

EXERCISE XXIV.

1. Divide 1554768 by 216. *Ans.* 7,198

2–7.

8514	9870	4370	2625000	6318	1248
22	15	21	350	27	24

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8. Divide 944 by 105, using factors. *Ans.* $8\frac{104}{105}$.

$105=5\times3\times7$

5)944		
3)188	4=1st rem.	4
7) 62	2=2d rem.	$2\times5=10$
8	6=3d rem.	$6\times3\times5=90$
	True rem.,	104

$944\div105=8\frac{104}{105}$.

111. To find the True Remainder.

RULE.—*Multiply each remainder by all the preceding divisors except that from which it arose, and add these products together for the true remainder.*

EXERCISE XXV.

Find the quotients of the following, using the factors of the several divisors:

1–5.	$\frac{8217}{35}$	$\frac{9591}{(2\times4\times9)}$	$\frac{801462}{108}$	$\frac{987654}{63}$	$\frac{926541}{81}$
6–10.	$\frac{94623}{105}$	$\frac{49684}{201}$	$\frac{349658}{42000}$	$\frac{8496}{28}$	$\frac{142693}{3600}$
11–15.	$\frac{468437}{7200}$	$\frac{34296}{56}$	$\frac{58734}{96}$	$\frac{1111111}{121}$	$\frac{343436}{84}$
16–20.	$\frac{594105}{105}$	$\frac{1494949}{49}$	$\frac{2565656}{56}$	$\frac{21493}{96}$	$\frac{277744}{77}$

112. Cutting off a figure from the right of a number brings the tens into the units' place, the hundreds into the tens' place, thousands into hundreds' place, and so on, thus reducing each order by one. If a figure be removed from the right, the number affected is divided by 10; if two figures are cut off, it is divided by 100; if three figures, by 1,000. The figures cut off must be considered as a *remainder.*

In the 8th example of the last exercise the divisor is 42,000. This equals $7\times6\times1000$.

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The first quotient is 349, with a rem. of 658; the second (dividing by 7) is 49, remainder 6; the third (dividing by 6) is 8, remainder 1.

The last remainder, multiplied by				7000 = 7000
The 2d	"	"	"	1000 = 6000
The 1st	"	"	"	= 658
			The true remainder is,	13658

113. To Divide by a number having Naughts at its right.

RULE.—*From the right of the dividend cut off as many figures as there are naughts at the right of the divisor. Divide by the rest of the divisor. Prefix the remainder to the figures cut off, for the true remainder.*

EXERCISE XXVI.

Perform the work indicated by the following:

1–3.	$\frac{376549281}{360\times480}$	$\frac{12345678}{420000}$	$\frac{89453}{2700}$
4–6.	$\frac{262800000}{45000}$	$\frac{273000}{18000}$	$\frac{3960180}{3500}$

7–15. Divide 6143598200 by these divisors: 150; 4800; 2100; 270; 5400; 1050; 31500; 14400; 5120; $(8\times8\times80)$.

16. How many dollars in 1564293 cents?

17. How many dollars in 14296000 mills?

18–21. In 4260 dimes? How many cents in 50 mills? In 42630 mills? In 270 mills?

22–24. How many dimes in 700 mills? In 72000 mills? In 4200 cents?

25–27. How many mills in $12; 140 dimes; in 12 cents?

114. When one product of several factors is to be divided by another product of several factors.

RULE.—*Strike out the factors common to both dividend and divisor, and seek the result with those which remain.*

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Example 1.—Divide $3\times8\times9\times17\times12$ by $9\times8\times7\times6$. Here 8 and 9 occur in both sets; besides, 12 contains the factor 6. Rejecting 8, 9, and 6 from each list, there remains the product $3\times17\times2$ to be divided by 7; *i.e.*, $102\div7$, which equals $14\frac{4}{7}$.

Example 2. $\frac{36\times49\times105}{24\times14\times21}$ Here 12 is a factor found in 36 and in 24; 7 is common to both 49 and 14; and 7 and 3 are both found in 105 and 21. These being struck out of the respective products, there remain $(3\times7\times5)\div(2\times2)=26\frac{1}{4}$. The problem, as performed, will appear thus:

$$\frac{\overset{3}{\cancel{36}}\times\overset{7}{\cancel{49}}\times\overset{5}{\cancel{105}}}{\underset{2}{\cancel{24}}\times\underset{2}{\cancel{14}}\times\underset{3}{\cancel{21}}}=\frac{105}{4}=26\tfrac{1}{4}.$$

Exercise XXVII.

(1) $\frac{16\times4\times5\times7}{8\times2\times35}$ (2) $\frac{4\times5\times3\times7\times8}{3\times5\times4\times14}$ (3) $\frac{20\times12\times48}{24\times4\times10}$

(4) $\frac{18\times16\times15\times4}{20\times15\times3}$ (5) $\frac{3\times17\times18\times24\times100}{34\times9\times36\times40}$

Divide 2146932 by 27.

Extended Process.	Contracted Process.
27)2146932(70000	27)2146932(79516
1890000	189
27)256932(9000	256
243000	243
27)13932(500	139
13500	135
27) 432(10	43
270	27
27) 162(6	162
162	162

Ans. 79516.

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Explanation of the Extended Process.—It will be observed that we have divided 5 separate parts of 2146932, the one after the other, and have added the results (Ax. VII.) It will also be seen that no more of the significant figures of the dividend were taken at a time than would give one significant quotient figure.

115. *General Rule for Division.*

Write the divisor on the left of the dividend, and draw a line between them. Take as many of the left-hand figures of the dividend as will contain the divisor, and write the number of times it is contained as the first figure of the quotient. Multiply the divisor by this quotient figure, and subtract the product from the part of the dividend taken, and to the remainder annex the next figure of the dividend.

Divide this number as before, and thus proceed till every figure of the dividend has been annexed.

Write the last remainder, if any, over the divisor, draw a line between them, and annex this quantity to the quotient.

NOTE *a.*—If, after having annexed a dividend figure to a remainder, the divisor be not contained, a **cipher** must always be placed in the quotient, and another dividend figure annexed, for a new partial dividend.

NOTE *b.*—If, after any subtraction, the remainder be greater than the divisor, a larger quotient figure should be written, and a new product obtained in place of the last.

EXERCISE XXVIII.

What are the quotients of the following examples?

(1.) $\frac{24242424}{365}$ (2.) $\frac{81428142}{1170}$ (3.) $\frac{5934}{65}$ (4.) $\frac{845693}{1700}$

(5.) $\frac{4596358}{125}$ (6.) $\frac{18423344}{1245}$ (7.) $\frac{7896500}{1895}$ (8.) $\frac{426397}{1172}$

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9. In a certain corn-field there are 426240 hills, arranged in 1280 rows. How many hills in a row?

10. An ear of corn had 768 kernels; each row contained 48 kernels. How many rows?

Divide each of the following dividends by each and every one of the divisors (64 quotients). Letter the results thus:

$$\frac{a}{i} \quad \frac{a}{j} \quad \frac{a}{k} \quad \frac{a}{l} \quad \frac{a}{m} \text{ etc.}; \quad \frac{b}{i} \quad \frac{b}{j};$$

11. *a.* 48189034	*i.* 392	The questions as numbered may be shown as follows:	
20. *b.* 605495736	*j.* 423		
28. *c.* 822656953	*k.* 937	11 $\frac{a}{i}$	12 $\frac{a}{j}$ 13 $\frac{a}{k}$
36. *d.* 552753652	*l.* 1034		
44. *e.* 352875520	*m.* 656	14 $\frac{a}{l}$	15 $\frac{a}{m}$ 16 $\frac{a}{n}$
52. *f.* 40832100	*n.* 639	17 $\frac{a}{o}$	18 $\frac{a}{p}$ 19 $\frac{b}{i}$
60. *g.* 232608375	*o.* 123		
68. *h.* 11029999176	*p.* 1113	20 $\frac{b}{j}$	21 $\frac{b}{k}$ etc.

116. *Questions on Division.*

What is meant by Division, dividend, quotient, divisor? In how many ways can you state the question, $12 \div 4 = ?$ Tell what Division teaches, making four different statements. What does a dividend equal? State the rule for dividing by less than ten. At which hand, or side, of the dividend is the work begun? Of what order will the first quotient figure be? When must a cipher be written in a quotient? Why are the principles and axioms of Division the same as those of Multiplication? What are the factors of the dividend? How will doubling a divisor affect the quotient? How else could the quotient be altered as much? What quotient would result if both dividend and divisor were doubled? What quotient if one-sixth as great a dividend and one sixth as great a divisor were used? How would a new quotient compare with one

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already found if the new dividend were 6 times as great as the first, and the new divisor were half as large as the first divisor? How, if the divisor were doubled and the dividend were halved? How, if the dividend were tripled and the divisor were but one-third of what it was before? How can a number be divided by a composite divisor? Repeat Axiom VIII. What principle is referred to when equal factors are stricken out of a composite dividend and composite divisor? State the proof of Division. Repeat the first rule for Division.

117. Bills and Accounts Current should generally give the date of each item and the date at which the account is presented:

BILL 1.

New York, Oct. 1, 1875.

Mr. Samuel Roberts,

Bought of Wm. Jenkins.

1875.						
May	4	12 yds. Flannel,	@	$ .28		
"	7	40 lbs. Sugar,	@	.17		
"	9	2 gals. Syrup,	@	1.12		
June	10	2 bbls. Flour,	@	8.50		
"	15	4 Scythes,	@	1.75		
July	17	3 lbs. Tea,	@	.87½		
Aug.	19	1 bu. Grass-seed,	@	4.25		
"	30	24 pairs Socks,	@	.50		
Sept.	4	45 lbs. Sugar,	@	.16		
"	9	12 lbs. Soda,	@	.05		
"	12	9 gals. Syrup,	@	1.10		
		Total,			$	72 98

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118. **BILL 2.**

LOCKPORT, June 10, 1874.

PAUL MANSON

To GEO. CLAY, *Dr.*

Jan.	1	To 51 yds. Muslin,	@	$0.22	
Feb.	4	" 12 Hdkfs.,	@	0.40	
"	12	" 2 prs. Boots,	@	5.50	
Mar.	9	" 4 yds. Cassimere,	@	3.25	
		Cr.			$
Mar.	9	By cash,			$ 12 00
		Balance due?			$

BILL 3.

MILWAUKEE, Dec. 31, 1875.

MR. SAMUEL VALENTINE

Bought of EDWARD BUSH.

1875.								
Mar.	1	3 cases Doeskins,	30	pcs.,	30	yds. ea.	@	$2.00
May	10	2 " f'cy Tweed,	28	"	32	"	@	1.20
June	5	4 " Satinet,	25	"	28	"	@	0.92
Sept.	10	6 " Cassimere,	30	"	31	"	@	2.25
Oct.	12	8 " Br'dcloth,	20	"	42	"	@	4.35
Nov.	12	9 " Flannel,	40	"	30	"	@	0.34
								$
		Cr.						
June	15	By cash,						$ 4400 00
Aug.	20	"						1600 00
Sept.	4	"						3000 00
		Balance due,						$

Received pay't,
EDWARD BUSH.

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BILL 4.

CHICAGO, May 10, 1876.

MESSRS. JOHN ROSE & Co.
Bought of ROBERT ROE.

May	1	20	dozen	printed Plates,	@	$2.00	.	
"	"	2	"	" Dishes,	@	10.00	.	
"	"	20	"	" Bowls,	@	1.00	.	
"	"	10	"	colored "	@	1.50	.	
"	"	10	"	" Mugs,	@	1.50	.	
"	"	10	"	fluted Tumblers,	@	3.00	.	
"	"	16	sets	printed Tea-sets,	@	0.50	.	
				Total, ...			$	148 00

Received payment,
ROBERT ROE.

BILL 5.

119. Transcript of Ledger.

MR. JAMES A. DRYASDUST,
In account with JOHN MURRAY.

1875.						
Jan.	21	2	Life of Washington,	@ $2.00	4 00	
"	"	2	Irving's Life Columbus,	@ 2.50	5 00	
"	22	5	Globe Shakespeare,	@ 4.50	22 50	
"	23	7	Grapes and Thorns,	@ 1.75	12 25	
Feb.	13	To 4 qrs. Cap, 20c.; 10 Note,			0.15	2 30
"	"	"	14 Ill. Hist. Ireland,	@ 4.50	63 00	
"	27	"	12 1st Ser. Ill. S.-School Lib.		3.00	36 00
Mar.	6	"	12 2d " " " "		3.00	36 00
"	13	"	12 3d " " " "		3.00	36 00
"	20	"	4 Brief Meditations,	@ 2.25	9 00	
		Cr.				
Feb.	6	By cash, ...				$ 79 00
"	27	"				88 00
Mar.	23	"				?
		Balance due, ...				$ 64 05

Received payment,
JOHN MURRAY.

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Bills of Account.

BILL 6.

BROOKLYN, January 1, 1872.

MR. PAUL MANSON,

In account with JOHN STEWART.

1871.							
May	1	To 28 yds. Calico,	@	$0.31	. .		
"	1	" 30 " Sheeting,	@	0.18	. .		
"	1	" 4 " blue Broadcloth,	@	7.00	. .		
"	30	" 2 " Alepine,	@	0.87½	. .		
June	4	" 2 " black Cassimere,	@	0.75	. .		
"	7	" 6 " Tweed,	@	0.75	. .		
July	2	" 4 " Linen,	@	0.87½	. .		
"	10	" 1 bbl. Rice, 270 lbs.,	@	0.14½	. .		
Sept.	1	" 4 bbls. Flour,	@	8.37½	. .		
"	30	" 28 gals. Molasses,	@	0.92	. .		
Nov.	9	" 45 lbs. Java Coffee,	@	0.48	. .		
						$	
		Cr.					
June	12	By 12 lbs. Butter,	@	$0.42	. .		
Oct.	10	" 2 tons Hay,	@	16.00	. .		
Nov.	1	" 42 weeks' pasturage,	@	0.75	. .		
"	10	" 120 lbs. Cheese,	@	0.22	. .		
Dec.	8	" 4 tons Hay,	@	18.00	. .		
						$	
		Balance due John Stewart, . . .				$	

120. *Signs and Abbreviations.*

a. The sign +, *plus,* is placed between numbers to be added.

b. The sign =, *equal,* is placed between equal quantities.

c. The sign ‾, *vinculum,* placed over numbers, and

d. The sign ( ), *parenthesis,* enclosing numbers within it, are used to denote that all the numbers included, or covered,

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are equally affected by whatsoever is outside of, or beyond, the parenthesis or vinculum. Thus: $\overline{2+3}\times\overline{5-2}=(2+3)\times(5-2)=5\times3$.

e. $\therefore$ signifies *therefore*, and precedes the conclusion of an explanation.

f. The sign —, *minus*, signifies that the second of two quantities is to be subtracted from the first, as $12-9=3$.

g. The sign $\sim$, *difference*, shows that a difference is to be found between two numbers. The larger may be written on the right. Thus $8\sim12=4$.

h. The sign $\times$ signifies that the numbers between which it stands are to be multiplied.

i. The sign $\div$ signifies that the first of the two numbers between which it stands is to be divided by the second.

j. The influence of $\times$ or $\div$ does not extend beyond the signs $+$ or —. Thus, $6+4\div2+18-3+27\div3=6+$"2"$+18-3+$"9," and is not equivalent to $(6+4)\div2+(18-3+27)\div3$. This last expression $=5+14$.

k. For such abbreviations as bu., pk., lb., see the *Tables*.

The questions for the above are obvious.

121. *Review Problems.*

GROUND RULES.

Exercise XXIX.

1. What number multiplied by 86 will give the same product as 163 by 430?

2. How long must a room be that is 180 inches wide to be as large as a room 176 inches by 742 inches?

3. If a man divide $9,281 among 17 men, and have remaining $373, what did each man get, if their shares were alike?

4. A father was 21 years old when his son was born. How old will the son be when his father is 50 years old? and what will the father's age be when the son is 50 years old?

Find the value of the following expressions (Art. 214.)

5. $15\times37153-73474-67152\div4+40734\times2$.

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6. 494871—94853+(45079—3177)—(54312—3987)—(1763 +231).

7. 379×462+324×561—(426+312)+4261÷84.

8. (7854—5236)×31416—(20843+277)÷5280.

9. 3914 + 3937 × 5760 — (7000+231)+(268+277274)÷ 1728.

10. 22182—21500÷5+(252458—39139).

11. (846+47+96)×25—20000÷25.

12. (14+7)×2+(256—25)÷21.

13. 1+3+95÷5—13+(46×3).

14. (896—176)÷144+(274+82)×(86—47+8).

15. 96+(144—97)×(86—47—18).

16. 16+25+33—2÷12+18—24.

17. A, B, and C "picked" 108 pears. Of these B and C picked 90, and A and C 51. How many of the 108 did each pick?

18. The "Iliad" contains 15683 lines, and the "Æneid" 9892 lines. If a boy read 85 lines a day, in how many days will he read through both?

19. In Prague, which contains a population of 12000, of every 12 persons 2 speak German only, 3 speak Tsech only, and 7 speak both German and Tsech. How many speak each language? How many both languages? How many German only?

20. The quotient arising from dividing 9281 by a certain number is 17, and the remainder is 373. Find the divisor.

21. The remainder of a division is 97, the quotient 665, and the divisor 91 more than the sum of both. What is the dividend?

22. Find the difference between the sum of 4715 added to itself 398 times, and the sum of 2017 added to itself 408 times.

23. A father bought for a Christmas-tree 43 articles at a dime each, 21 at 7 for a dime, 51 at 3 for a dime, 7 for 25 cents, 28 at 4 for 25 cents, and 17 at 5 for 50 cents. How much did he spend in all?

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24. Carolina rice being 5*d.* per lb., and 44*s.* per cwt.; East India rice, 4*d.* per lb., and 35*s.* per cwt.; and broken E. I. rice, 2½*d.* per lb., and 22*s.* per cwt.; find how much is gained by buying a cwt. of each at the wholesale price of each, and selling it at retail, reckoning the cwt. at 112 lbs.

25. If the two sides of a field are each 63 rods in length, and the two ends 42 rods each; allowing 9 rails to the rod, how many rails will be required to fence the field?

26. Two persons, A and B, start at the same time from places 375 miles apart, and travel toward each other—A at the rate of 37 miles a day, and B at the rate of 43 miles a day. At the end of the second day, how far is A from B, and how far is each from the other's starting-point?

27. The annual deaths in a town being 1 in 45, and in the country 1 in 50, in how many years will the number of deaths out of a town population of 18675, and a country population of 79250, number 10000?

28. The product of the ages of two boys, increased by the age of a third boy 11 years old, is 167; and one of the two is 13 years of age. What is the age of the other?

29. A farmer's crop of wheat brought him $650; his barley, $205; his corn, $97; his oats, $75. He paid his laborer $13 a month for 9 months, bought a pair of horses for $190, and paid for farming utensils and repairs $65. How much of his income remained?

30. A farmer paid $7950 for a farm. He sold 50 acres at $40 an acre, and found that if he sold the remainder at $35 an acre he would neither gain nor lose. How many acres did he purchase?

31. A man bought 15 cows at $35 each, and 23 cows at $38. What would he gain if he should sell them all at $40 each?

32. A merchant collected from A $250; from B, $318; and from C as much as from A and B both, lacking $25. He then expended the whole amount collected for cloth at 8½ dollars a yard. How many yards did he buy?

33. A man dying, left an estate worth $4500. His debts

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amounted to $524; the remainder was divided equally among his 4 children. Required the share of each.

34. A bought 45 bbls. of flour at $6 a bbl.; he sold 32 bbls. at $8, 5 bbls. at $9, and the remainder at $7. Required his gain.

35. Four boys agreed to share equally what chestnuts they might gather. A found 257, B collected 274, C got 195, and D 378. What was the share of each?

36. Thomas sold 16 *Tribunes*, at 4 cents each; Richard, 46 *Posts*, at 5 cents each; Henry, 96 *Suns*, at 2 cents each. They shared the money obtained equally. How much did Henry get?

37. A farmer sold 12 tons of hay at $16; 900 lbs. of pork at $6.50 per hundred. He then bought 15000 shingles at $7 per *M*, and a quantity of lumber for $125. How many lbs. of nails at 8 cents a lb. could he have purchased for the rest of the money?

38. A and B are together, and set off travelling in the *same* direction, A at the rate of 37 miles a day and B at the rate of 45 miles. How far is B in advance at the end of the 12th day? How far apart would they be if they had travelled in *opposite* directions?

39. What number is that which, being multiplied by 24, the product divided by 10, the quotient multiplied by 2, 32 subtracted from the product, the remainder divided by 4, and the quotient diminished by 8, the remainder shall be 2?

40. From $126+(16+4)\times 2$ take $\frac{48+34\times 6}{17-5}$

41. Required the quotient of $162\times 0\times 41\div 35\times 81$.

42–44. A load of wheat sold by a farmer weighed 2700 lbs. At $1.20 a bushel of 60 lbs., what was it worth? How many bbls. of good flour, if a bushel will yield 45 lbs.? Required the value of the flour at 4 cts. a lb.? 196 lbs.=1 bbl.

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PROPERTIES OF NUMBERS.

122. Numbers are sometimes classified as Prime and Composite.

123. A **prime number** has no other exact divisor than itself and unity.

124. A **composite number** is the product of two other integers.

125. The **factors** of a number are such other numbers as when multiplied will produce it.

126. A **factor** is sometimes called a **divisor** or **measure** because it will exactly divide some composite number.

127. The **prime factors** of a number are the prime numbers which produce it. 3, 5, and 7 are the prime factors of 105.

128. A **factor** is **common** to two or more numbers when it is a factor or measure of each of them.

129. The greatest common divisor, G.C.D., of two or more numbers is the greatest number which will exactly divide each of them. 12 is the G. C. D. of 24, 36, 48, and 60. It is their greatest common factor.

130. 2 will exactly divide any **even** number, 4 will measure any number whose **last two digits** will contain it. 5 measures every number ending in 0 or 5. If either 6 or 9 is exactly contained in the **sum of the digits of a number,** it is exactly contained in that number. 8 will measure any number whose last three figures contain it. Any number of 4 figures consisting of two like digits, separated by two ciphers, will exactly contain 7, 11, and 13.—*e.g.*, 1001.

131. Any number is exactly contained in its **multiple.**

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132. An **exponent**, or index, is a small figure written at the right and above a number to show how many times the number is taken as a factor. Thus, in the expression $32=2^5$, 5 is an exponent. $288=2^5\times3^2$ or $2\times2\times2\times2\times2\times3\times3$.

A SHORT TABLE OF PRIME NUMBERS.

1	11	23	41	59	73	97	109
3	13	29	43	61	79	101	113
5	17	31	47	67	83	103	127
7	19	37	53	71	89	107	131

The Greatest Common Divisor.

133. TO FIND THE GREATEST COMMON DIVISOR (G. C. D.)

RULE.—*Divide the greater of the two given numbers by the less; then the divisor by the remainder; then divide the last divisor by the last remainder, and continue till there is no remainder. The last divisor will be the G. C. D.*

If there be more than two numbers whose G. C. D. is required, find the G. C. D. of either two of the numbers; then the G. C. D. of this exact divisor and one more of the numbers; then the G. C. D. of this last measure, and still another of the numbers, until every number has been considered.

Generally, the very last division mentioned under the rule may be omitted, as the result is usually obvious, as in finding the G. C. D. of 602 and 1032.

In this example it is plain that the last mechanical division of 172 by 86 might have been omitted, as a glance would show that there would be no remainder.

602)1032(1
602
430)602(1
430
172)430(2
344
86)172(2
172
000

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Methods of securing Accuracy.

134. If any sagacious man of business were asked, "What is the first requisite in an accountant?" he would certainly answer, "**Accuracy.**"

A teacher should, as a general rule, attach no value to inaccurate work, and thus cultivate this prime virtue. He should teach the pupil the best methods of testing and verifying his own work.

135. *Strict Proof is the* **Opposite Process.**

The proof of Subtraction is Addition. Division is the proof of Multiplication.

136. If **two distinct processes** have led to the same result, the work may be considered as proved.

137. Repetition is generally considered as proof, especially if the work be slightly varied; as, for instance, the addition of a column by beginning the second addition at the top.

138. CASTING OUT NINES

There is one reasonably certain method which is of special utility because of the facility with which it can be applied. This method is based on the peculiar property of the number 9, which peculiarity may be stated thus: *If any number be divided by* 9, *and the sum of its digits be divided by* 9, *the remainders will be the same.*

Explanation.—(*a*) As 9 is the highest figure, and as any number of units less than 9 is not divisible by it, such small numbers may be considered as remainders. If 10, 100, 1000, etc., were divided by 9, there would plainly be a remainder 1; if 20, 200, 2000, etc., a remainder 2; if 30, 300, 3000, etc., a remainder 3; if 70, 700, 7000, etc., a remainder, 7, etc. It cannot fail to be apparent what remainders will arise when the several orders of a number are divided by 9.

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Explanation of Example 1.—If in the first number, 3000, 400, 80, 7 be divided separately by 9, the several remainders will be 3, 4, 8, and 7. If the second number be divided in the same way, the remainders will be 4, 3, 5, and 1. If the third number be so divided, the remainders will be 8, 4, 7, and 5. And if the sum of the remainders of any one of these numbers be divided by 9, the final remainder, if any, will evidently be the same as that obtained by dividing the number in the first place by 9; for, in both cases, the final remainder is the excess over the full number of 9s contained in the given number or dividend. The sum of the quotients will be the same as the quotient of the sum; and, as we know beforehand that these two results are equal, we may dispense with all this division, and deal with the several remainders which are now simply so many units capable of being added in any direction—horizontally or vertically. Thus the first number, 3487, may be regarded as $3+4+8+7$ *units* (*a*), the figures having lost, by the previous supposed division by 9, their local value.

Ex. 1.—1st No. 3487 . . . . +4
2d No. 4351 . . . . +4
3d No. 8475 . . . . +6
16313+5 9)14(1
5

The whole operative **proof of Addition** by casting out the 9s is (1) *to find the sum of the units taken horizontally, and the sum of the units taken vertically, and, dividing each sum by* 9, *to see if both remainders are alike.*

Or, more briefly, (2) *it is to find the sum of the digits of the numbers, then the sum of the digits of this sum, and so on until we arrive at a sum of one digit.*

In problem 1 we may *say:*

7, 15, 22, remainder 4. 3487 4
7, 12, 13, remainder 4. 4351 4
12, 19, 24, remainder 6. 8475 6
10, 14, remainder 5. 16313 . . . . 14–5.

Sum: 7, 10, 11, 14, remainder 5.

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Or, by the second method:

7, 15, (6), 13, (4) 4
7, 12, (3), 4, ... 4 } 5.
12, (3), 10, (1), 6 6
10, (1), 2, 5, 5=5.

The numbers in the parentheses are formed by adding the digits of each sum before proceeding to add in the next figure. Thus, in 8475, 8+4=12 (3) [for 1+2=3]; then 3+7=10 (1); and, finally, 1+5=6.

The pupil should read the following work (understandingly, of course) until he can go on easily by himself.

Example 1.—To find the "9-over" (or remainder after dividing by 9) of 6841, we begin: 6, 14 (here we reduce 14 to 5 by adding 1 and 4), 5, 9 (drop the nine), 1. *Ans.* 1.

Thus the wording really is: 6, 14, 5, 9, 1. *All 9s, and all pairs of numbers which together make* 9, *are at once passed over.*

Example 2.—78563. Wording: 7, 15, (6), 11, (2). The 6 and 3 are passed over, as they obviously form 9.

Example 3.—675432. Wording (not mentioning the first figure): 13, 4 (skip the figures 5 and 4), 7, 9. *Ans.* 0.

Example 4.—8357924583262. Wording: 11, (2), 7, 14, (5) (skip 9), 7, 11, (2), 7, 15, (6) (skip 3 and 6). *Ans.* 2.

In this last "wording," the **2**, the **5**, the **2**, and the **6** were formed by adding the digits of 11, 14, 11, and 15. For an exercise, take problems in addition.

Proof of Multiplication by casting out the 9s.

RULE.—*Cast the* 9*s out of the multiplicand and also out of the multiplier. Cast the* 9*s out of the product of the "overs," and if the final "over" equals that found by casting the* 9*s out of the product, the work is probably right.*

Example 1.—Is 920685×7098=6535022130? FACTORS.—Wording: 8, 16, 7, 12, **3**, then 15, **6**, then 6×3, 18, (9), **0**. PRODUCT.—Wording: 11, 2, 5, 10, 1, 3, 5, 6, 9, **0**. Results alike; product right.

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To prove Division. *Consider the dividend as the product of the divisor and quotient. The remainder must first be subtracted from the dividend before adding its digits.*

Example 1.—$3067430 \div 8427 = 364\frac{2}{8427}$

The "over" of 3067430—2, or 3067428, is **3**.

The "over" of 8427 is 3; of 364 is 4;

The "over" of the product of the "overs" 3 and 4 or 12 is 3, which corresponds with the over of the dividend.

Least Common Multiple.

139. When a number is multiplied, the product is its multiple. If 4 be multiplied by 3, the product is not only a multiple of 4, but is also a multiple of 3; in other words, 12 is a multiple *common* to both these numbers.

140. A Common Multiple of several numbers contains each of the numbers as a *factor*.

MULTIPLE TABLES.

2 4 6 8 10 **12** 14 16 18 20 22 **24**..***36***
3 6 9 **12** 15 18 21 **24** 27 30 33 ***36***
4 8 **12** 16 20 **24** 28 32 ***36***
6 **12** 18 **24** 30 ***36***

It will be observed that 12, 24, and 36 are each common multiples of 2, 3, 4, and 6. In each one, these four small numbers are found as factors.

A multiple can of course be divided by its factors, as division is simply the "undoing" of multiplication.

12 is the *least* number in which each of the numbers 2, 3, 4, and 6 is exactly contained, and is therefore called their *Least Common Multiple.*

Now, the product of 2, 3, 4, and 6 is 144. The Least Common Multiple (abbreviation L. C. M.) need not necessarily be the product of *all* the numbers. The multiple of 6 is, of course, a multiple of 2 or of 3, and the multiple of 4 must be a multiple of 2. When a number comprises all the prime

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factors of another, it is a multiple of that other. The different prime factors of 4 and 6 are 2 twos and 1 three. The product of these factors 2, 2, 3, is therefore the L. C. M. of 2, 3, 4, and 6.

141. To find the L. C. M.

Rule.—*From the list of numbers given, omit every one which is a divisor of either of the others. Write those remaining in a line of dividends. Divide by any prime number that is exactly contained in two or more of them and set the quotients and undivided numbers in a line beneath. Omitting again all such as may be divisors of others in the line, proceed with the second line as with the first, and continue to divide each line until no prime number is exactly contained in more than one of the numbers. Finally, take the continued product of the divisors and undivided numbers for the L. C. M.*

1. Find the L. C. M. of 18, 2, 3, 4, 75, 16, 42, 36, 385, 48.

5	385	75	48	42	36
3	77	15	48	42	36
7	77	5	16	14	12
2	11	5	16	2	12
2	11	5	8	1	6
	11	5	4	1	3

L. C. M. $=5\times3\times7\times2\times2\times11\times5\times4\times3=277200$.

The L. C. M. may also be ascertained by resolving each of the principal numbers into its prime factors and then taking each of these factors as a factor as many times as it is found in either of the given numbers. The product will be the L. C. M.

In factoring, the sub-numbers are omitted, as in the Rule above.

The L. C. M. is important in the addition and subtraction of fractions.

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EXERCISE XXX.

Find the L. C. M. of each of the following sets:

2.	7	6	9	42	30	630	13.	5	19	125	16	74	
3.	36	48	80	144	96		14.	13	14	91	6	4	32
4.	9	14	35	28	18		15.	6	9	3	81	51	17
5.	6	8	10	12	14		16.	3	6	5	28	42	16
6.	3	5	7	9	11		17.	17	35	14	42	21	70
7.	4	8	16	32	64		18.	8	11	32	44	15	45
8.	42	36	21	20	18	10	19.	3	5	13	25	39	46
9.	20	48	50	72	36		20.	3	9	21	14	28	7
10.	5	15	25	35	45		21.	35	45	55	70	90	110
11.	1	2	3	4	5	9	22.	16	26	52	13	32	8
12.	4	6	10	12	16	32	23.	7	17	27	37	47	57

24. Required the smallest quantity which can be exactly measured by pails of the following capacities, viz., 4, 6, 8, 10, and 14 quarts.

25. A man found that each of several pails was an exact measure of the contents of a certain vat. The vessels would hold respectively 1 pint, 3 quarts, 3 gallons, and 5 pecks; how many quarts would fill the vat?

26. Required the smallest sum which will exactly buy an equal number of doves, ducks, dogs, and donkeys; doves being worth 35 cents, ducks 75 cents, dogs $4.25 cents, and donkeys $10.50 cents each. *Ans.* $178.50.

27. Find the L. C. M. of 1001, 49, and 77.

28. Find the L. C. M. of 174, 58, and 36.

29. Find the L. C. M. of 4004, 7, 11, and 13.

30. Find the L. C. M. of 3003, 6, 7, 11, and 13.

31. Find the L. C. M. of 4536, 81, and 202.

32. Find the L. C. M. of 8, 4152, and 519.

142. NOTE.—The L. C. M. of *two* large numbers may be found by dividing either one by their G. C. D. and multiplying the quotient by the other number—*e. g.*, G. C. D. of 14758 and 9263 is 157; $14758 \div 157 = 94$; $94 \times 9263 = 870722$.

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Complements of Numbers.

If it were required to multiply 4624 by 999, the result might be obtained thus: 4624000 — 4624 = 4619376. The complement of a number is the difference between it and a unit of the next higher order: 1 is the complement of 9; 3 is the complement of 97; 25 is the complement of 75. The product of a multiplicand and the complement of a multiplier may often be subtracted from a product otherwise too large. Thus, $878 \times 9998 = 8780000 - (878 \times 2)$; $5280 \times 975 = 5280000 - 528000 \div 4$, or $132000 = 5148000$.

143. *Questions.*

Define these arithmetical terms, viz.: Integer, Fraction, Odd Number, Factor, Measure, Composite Number, Prime Number, Common Factor, Common Divisor, G. C. D., Measure, Multiple, L. C. M., Proof. State how Addition, Subtraction, and Multiplication may be proved by "casting out the 9s." What is the L. C. M. of 2, 3, 4, and 6? State the rule for finding the G. C. D. State the rule for finding the L. C. M.

144. *Questions for General Review.*

Define the term Unit, Number, Arithmetic, Notation, Numeration, Order, Decimal, Tens, Period, and Digit. Name the first eight periods. Name the 3d Order; the 4th; the 10th; the 14th; the 17th; the 2d; the 5th; the 9th; the 18th; the 7th; the 17th. Why must ciphers be written in certain cases? In what order do hundreds stand? Millions? Hundreds of thousands? Thousands? How many figures are required to write 41 millions? 213 billions? Define *Plus.* What is meant by Sum? Pronounce 7, and add by 6s to 61. Pronounce 3, and add by 4s to 39. Pronounce 11, and add by 8s to 107. Recite Axiom I. Define "Concrete Number." Recite Axiom II.; Axiom III. Define "Tenth"; "Fourth." Recite the Table of U. S. money. Are accounts

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kept in mills, dimes, and eagles? What is meant by a census? A city? A county? Define Subtraction, Subtrahend, Minuend, Remainder, *Minus*. Repeat Axiom IV. What is the abbreviation for Bushels? Pecks? Dimes? Dollars? Quarts? Pounds sterling? Shillings? Pence? Define Multiplication; Factor; Prime Number; Composite Number; Product; Multiplicand. Show that a product of two numbers is the same if either be taken as a multiplier. Define Division; Dividend; Measure; Common Measure; G. C. D.; Multiple; Common Multiple; L. C. M. State this problem, viz.: "Multiply the difference between 78 and 49 by 3; from the product subtract the difference between 128 and 124; to the remainder add 43, and divide the sum by 6." Find the Greatest Common Divisor (G. C. D.) of 849 and 1132, also their L. C. M. How is Division proved? How Multiplication? How are the 9s cast out in proving Addition? How in proving Multiplication? Division? Repeat Axiom, VI. What are the four definitions of Division? Repeat Axiom VII. Repeat the four *principles* of Division, and illustrate them.

COMMON FRACTIONS.

Section VII. General Theory of Fractions.

145. In dividing 7 by 3, a remainder of 1 appears. As this 1 is a pure number, we cannot actually divide it as if it were an apple, but can only represent its division thus: $\frac{1}{3}$. Thrice $\frac{1}{3}$ is, of course, the unit left over. The expressions $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ are similar. Each of these ones, or fractional units, derives its

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name from the number below the line. If it were required to divide 3 by 5 (represented thus: $\frac{3}{5}$), the result would be called three-fifths, as if the division *had actually been performed.*

If one were allowed to take $\frac{1}{2}$ of 6 apples, he might take 3 undivided apples, or else take 6 half-apples. One-half of 6 equals six halves of 1.

146. A **Fraction,** or **Fractional Number,** expresses one or more equal parts of a unit.

147. A Fraction is formed of two numbers, one of which, called the **Numerator,** shows how many parts are taken, and the other, called the **Denominator,** names the parts and shows what is the divisor of the unit.

148. The Numerator is written over or at the left of the Denominator.

149. The numbers used to express a fraction are called its **terms.** A short **line** should separate the terms.

150. If 1 be divided into three equal parts, one of the parts is called a third. If two parts are to be written, the expression is $\frac{2}{3}$. If we would represent 2 as divided into 3 equal parts, we would write an expression like the last, viz., $\frac{2}{3}$.

151. One-third of 2 equals two-thirds of 1. The Numerator, considered as a whole, is called the **"Unit of the Fraction."** One of the equal parts into which the Numerator is divided is called a **Fractional Unit.** Thus, in the fraction $\frac{2}{3}$, 2 is the Unit of the Fraction, and $\frac{1}{3}$ is a Fractional Unit. (Young pupils may omit this paragraph.)

152. *Every fraction expresses a problem in Division,* the *numerator* being the *Dividend,* and the *denominator* the *Divisor.* The *Quotient* equals the **value of the Fraction.**

153. A number less than 1 must be fractional, and when a divisor is not exactly contained in its dividend, the result must be a fraction.

154. Fractions are usually classified as follows: *Simple, Complex, Compound, Proper,* or *Improper.*

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155. Both terms of a **Simple Fraction** are integers, as $\frac{3}{5}$, $\frac{5}{3}$.

156. One or both terms of a **Complex Fraction** may be fractional, as $\frac{2}{\frac{3}{5}}, \frac{\frac{5}{2}}{4}, \frac{\frac{3}{4}}{\frac{5}{6}}$.

157. A **Compound Fraction** is a fraction of a fraction, as $\frac{2}{3}$ of $\frac{4}{5}$; or a fraction of an integer, as $\frac{2}{3}$ of 1

158. A **Proper Fraction** expresses less than 1, as $\frac{2}{3}$.

159. An **Improper Fraction** expresses 1 or more than 1, as $\frac{2}{2}$, $\frac{5}{3}$, $\frac{8}{3}$.

160. If a fraction is annexed to an integer, the expression is called a **Mixed Number,** as $3\frac{1}{3}$, read 3 (pause) and $\frac{1}{3}$.

161. Similar Fractions have a common fractional unit—that is, the parts have the same name. $\frac{2}{3}$, $\frac{4}{3}$, $\frac{7}{3}$ are similar fractions.

162. As the quotient obtained by dividing any number by 1 must just equal the number divided, *any integer may be represented by a fraction whose numerator is the integer and whose denominator is* 1. Thus, $\frac{4}{1}=4$; $\frac{7}{1}=7$.

163. When the denominator of a fraction is 10, 100, or any product whose factors are 10, it is called a **Decimal fraction.** The *numerator only is written.* It is preceded by a *point. The denominator is always understood to be* 1, *with as many naughts annexed as there are figures in the numerator.* If both terms are *expressed,* the quantity is called a Common Fraction.

164. In dividing the numerator of a common fraction by the denominator, if there be a remainder, one or more naughts may be annexed and the division continued. The result will be a *decimal fraction.*

165. The denominator of a Common Fraction may be *any* number; that of a Decimal Fraction must be **10,** or a product each of whose factors is 10.

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166. *Exercise XXXI.* (Oral.)

When a number is divided into two equal parts, what is one part called? What is $\frac{1}{2}$ of 2? Of 4? Of 6? Of 12? Of 18? Of 24? How is one-half of a number obtained? One-third? One-fourth? One-fifth? One-tenth? One-twentieth?

What is a third of 6? Of 21? 9? 12? 24? 18? 21? 15?

What is a fourth of 8? Of 20? 12? 16? 24? 40? 36? 32?

What is a fifth of 10? Of 20? 30? 15? 25? 40? 35? 45?

What is an eighth of 8? Of 80? 16? 32? 48? 24? 40? 56?

What is a tenth of 10? Of 40? 100? 90? 70? 20? 60? 80?

What is a seventh of 14? Of 70? 63? 21? 56? 42? 49?

What is a fourth of anything? A third?

If an apple is worth 3 cents, what part of it is worth 1 cent? 2 cents?

How many half-dollars in \$4? In \$6? \$13?

How many quarters in \$7? In \$9? \$11?

How many thirds in two units? In ten?

If a cord of wood costs \$8, what costs $\frac{1}{2}$ of a cord? $\frac{1}{4}$ of a cord? $\frac{1}{8}$ of a cord? 2 cords? $2\frac{1}{2}$ cords?

Analysis.—(*a*) Find cost of 2 cords; (*b*) cost of $\frac{1}{2}$ a cord; (*c*) add the two values. Thus: 2 cords will cost \$16; $\frac{1}{2}$ cord, \$4; \$16+\$4=\$20, or the cost of $2\frac{1}{2}$ cords at \$8 a cord.

What is meant by 2 fourths? *Ans.* Two of the four equal parts into which a number is divided.

What is meant by $\frac{2}{4}$? $\frac{2}{5}$? $\frac{2}{15}$? $\frac{3}{4}$? $\frac{5}{6}$? $\frac{3}{5}$?

What is $\frac{1}{2}$ of 6? $\frac{2}{2}$? $\frac{1}{6}$ of 12? $\frac{3}{6}$? $\frac{5}{6}$?

3 is $\frac{1}{2}$ of what number?

Solution.—All things have 2 halves; if $\frac{1}{2}$ is 3, the other half must also be 3; and both halves, or the whole number, must be 3 and 3, or 6.

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4 is $\frac{1}{2}$ of what number?

Solution.—If 4 is one half, 2 halves must be twice 4, or 8; therefore, 4 is one half of 8.

4 is $\frac{1}{3}$ of what number? $\frac{1}{5}$ of what number? $\frac{1}{6}$?
5 is $\frac{1}{2}$ of what? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{9}$? $\frac{1}{10}$? $\frac{1}{12}$?
6 is $\frac{1}{3}$ of what? $\frac{1}{4}$? $\frac{1}{6}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{11}$?
4 is $\frac{2}{3}$ of what number?

Solution.—If *two* thirds of a number be 4, *one* third of the number is half of 4 or 2, and if *one* third is 2, *three* thirds or the entire number must be 3 times 2 or 6.

3 is $\frac{3}{5}$ of what number?

Solution.—If 3 be $\frac{3}{5}$ of a number, $\frac{1}{5}$ of 3, or 1, is $\frac{1}{5}$ of the number; and $\frac{5}{5}$ of it is 5 times 1, or 5; ∴ 3 is $\frac{3}{5}$ of 5.

4 is $\frac{4}{7}$ of what number?

Solution.—If 4 be $\frac{4}{7}$, 1 is $\frac{1}{7}$; and $\frac{7}{7}$, or the entire number, must be 7.

6 is $\frac{3}{4}$ of what number?

If 6 is $\frac{3}{4}$, *one* fourth is $\frac{1}{3}$ of 6 or 2, and *four* fourths, or the entire number, must be 8. Therefore 6 is $\frac{3}{4}$ of 8.

6 is $\frac{3}{4}$ of what number?

Second Solution.—If 6 is $\frac{3}{4}$ of a number, $\frac{1}{3}$ of 6, or 2, is $\frac{1}{4}$, and $\frac{4}{4}$, or the whole number, is 4 times 2, which is 8.

4 is $\frac{2}{5}$ of what number? $\frac{2}{7}$? $\frac{2}{9}$? $\frac{2}{11}$?
12 is $\frac{3}{4}$ of what number? $\frac{3}{5}$? $\frac{3}{7}$? $\frac{4}{5}$? $\frac{4}{9}$?
12 is $\frac{6}{7}$ of what number? $\frac{6}{10}$? $\frac{8}{9}$? $\frac{3}{10}$? $\frac{4}{11}$?
21 is $\frac{7}{10}$ of what number? $\frac{7}{12}$? $\frac{7}{15}$?
What is $\frac{2}{5}$ of 10? Of 15? Of 25? Of 45?
What is $\frac{3}{4}$ of 12? Of 16? Of 24? Of 36?
What is $\frac{2}{3}$ of 6? 18? 21? 15? 24?
What is $\frac{1}{2}$ of 4? $\frac{1}{2}$ of 4 lbs.? $\frac{1}{2}$ of 4 dimes?
What is $\frac{1}{3}$ of 6 lbs.? Of $\frac{6}{8}$? $\frac{6}{18}$? $\frac{6}{10}$?
What is $\frac{2}{3}$ of 6 lbs.? Of 6 dimes? Of $\frac{3}{5}$?

Solution.—*One* third of $\frac{3}{5}$ is $\frac{1}{5}$, *two* thirds of $\frac{3}{5}$ is twice as much, or twice $\frac{1}{5}$ which are $\frac{2}{5}$.

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What is $\frac{2}{5}$ of $\frac{5}{6}$?

Analysis.—(*a*) Find $\frac{1}{5}$ of $\frac{5}{6}$; (*b*) then $\frac{2}{5}$ of the same.

What is $\frac{3}{7}$ of $\frac{7}{10}$? Of $\frac{14}{10}$? $\frac{21}{12}$? $\frac{28}{30}$?
What is $\frac{4}{5}$ of $\frac{5}{7}$? $\frac{10}{7}$? $\frac{10}{12}$? $\frac{15}{8}$? $\frac{15}{6}$?

Section VIII. Reduction of Fractions.

We have seen that $\frac{2}{2}$, or 1, equals $\frac{4}{4}$, and that $\frac{1}{2}$ is equal to $\frac{2}{4}$; also that $2\frac{1}{2}$ is equal to $\frac{5}{2}$, and $1\frac{1}{4}$ to $\frac{5}{4}$. In these instances we might substitute either of the expressions for its equivalent, as $\frac{5}{2}$ for $2\frac{1}{2}$, $\frac{2}{4}$ for $\frac{1}{2}$, etc.

167. The Reduction of a Fraction consists in finding such other terms as will represent an equal number.

168. To reduce an Improper Fraction to the form of an Integer, or of a Mixed Number.

RULE.—*Divide the numerator by the denominator; set the remainder, if any, over the divisor, and annex this fraction to the quotient.*

EXERCISE XXXII.

Reduce the following to whole or mixed numbers:

1. $\frac{31}{10}$.	2. $\frac{28}{3}$.	3. $\frac{45}{6}$.	4. $\frac{21}{8}$.	5. $\frac{39}{5}$.
6. $\frac{43}{9}$.	7. $\frac{52}{7}$.	8. $\frac{84}{9}$.	9. $\frac{75}{8}$.	10. $\frac{723}{4}$.
11. $\frac{21}{8}$.	12. $\frac{312}{75}$.	13. $\frac{7296}{81}$.	14. $\frac{312}{25}$.	15. $\frac{85}{7}$.
16. $\frac{39}{5}$.	17. $\frac{24}{11}$.	18. $\frac{19}{7}$.	19. $\frac{36}{9}$.	20. $\frac{42}{14}$.
21. $\frac{33}{3}$.	22. $\frac{47}{11}$.	23. $\frac{111}{11}$.	24. $\frac{84}{11}$.	25. $\frac{96}{11}$.
26. $\frac{564}{29}$.	27. $\frac{455}{76}$.	28. $\frac{12561}{356}$.	29. $\frac{256}{64}$.	30. $\frac{113}{15}$.

169. How many fifths in $2\frac{1}{5}$?

$1=\frac{5}{5}$; $2=\frac{10}{5}$; $\frac{10}{5}+\frac{1}{5}=\frac{11}{5}$; or, as $(2\times5)+(\frac{1}{5}\times5)=11$, therefore $\frac{2\frac{1}{5}\times5}{1\times5}=\frac{11}{5}$.

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To Reduce a Mixed Number to an Improper Fraction.

Rule.—*Multiply the integer by the denominator, add in the numerator, and write the sum over the original denominator.*

Exercise XXXIII.

Reduce to improper fractions:

1. $15\frac{7}{10}$; $12\frac{7}{12}$; $2\frac{4}{17}$; $3\frac{8}{125}$; $100\frac{7}{10}$; $9\frac{3}{5}$; $4\frac{2}{13}$;
8. $27\frac{2}{13}$; $44\frac{9}{59}$; $1000\frac{7}{50}$; $6\frac{1}{4}$; $16\frac{3}{5}$; $21\frac{1}{1000}$; $18\frac{1}{3}$;
15. $192\frac{1}{11}$; $8\frac{5}{16}$; $9\frac{1}{19}$; $82\frac{7}{51}$; $9\frac{1}{17}$; $3\frac{14}{70}$; $9\frac{7}{100}$;
22. $44\frac{9}{159}$; $102\frac{9}{200}$; $1001\frac{1}{11}$; $11\frac{1}{1001}$; $15\frac{1}{5}$; $5\frac{1}{15}$; $22\frac{7}{16}$.

Remark.—$\frac{1}{2}$ a quart=1 pint; $\frac{1}{8}$ peck=1 quart; $\frac{1}{12}$ of a shilling=1 penny; $\$\frac{1}{10}$=1 dime; $\frac{1}{4}$ gallon=1 quart; $\frac{1}{4}$ of a bushel=1 peck.

23. Reduce to pints $2\frac{1}{2}$ quarts; $4\frac{1}{2}$ quarts; 6 quarts.
26. Reduce to quarts $3\frac{1}{8}$ pecks; $4\frac{1}{8}$ pecks; $8\frac{3}{8}$; $9\frac{7}{8}$.
30. Reduce to pence $4\frac{3}{12}$ shillings; $5\frac{1}{12}$; $6\frac{7}{12}$; $9\frac{8}{12}$.
34. Reduce to dimes $\$4\frac{3}{10}$; $\$7\frac{7}{10}$; $\$9\frac{6}{10}$; $\$45\frac{5}{10}$.
38. Reduce to quarts $4\frac{3}{4}$ gallons; $5\frac{1}{2}$; $7\frac{1}{4}$.
41. Reduce to gills $21\frac{1}{4}$ pints; $30\frac{1}{2}$; $15\frac{3}{4}$; $17\frac{1}{2}$.
45. Reduce to pecks $16\frac{1}{4}$ bushels; $20\frac{1}{2}$; $19\frac{1}{4}$; $24\frac{3}{4}$.
49. Reduce to furlongs $3\frac{1}{2}$ miles; $12\frac{1}{8}$; $16\frac{2}{16}$; $13\frac{5}{8}$.

General Principles of Fractions.

Remembering that Division is the process of separating a number into equal parts, and that the numerator and denominator stand to each other in the relation of dividend and divisor, we deduce the following principles:

170. I. *To multiply a fraction,* { *Multiply the numerator, or* / *Divide the denominator.*

171. II. *To divide a fraction,* { *Divide the numerator, or* / *Multiply the denominator.*

172. III. *To merely change its form,* { *Multiply both terms, or* / *Divide both terms.*

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173. The effect of changing the terms of a fraction may also be stated as follows: A fraction may be made larger by increasing either the number of its parts or the size of its parts.

It may be made smaller by lessening the *number* of its parts or by decreasing the size of its parts.

Its value will remain unaltered if, when the number of parts is increased, their size is diminished in the same proportion.

Its value will not be changed if, when the number of parts is decreased, their size is increased in the same ratio or degree.

174. Fractions to their Lowest Terms.

From General Principle II. we deduce the following:

RULE.—*Divide both terms by their G. C. D.*

1. Reduce $\frac{105}{490}$ to its lowest terms: $\frac{35)105}{35)490}=\frac{3}{14}$.

Steps.—*a.* Find the greatest common divisor.
b. Divide the numerator by the G. C. D.
c. Divide the denominator by the G. C. D.

(*a*) $490\div105=4\frac{70}{105}$; $105\div70=1\frac{35}{70}$; $70\div35=2$.
(*b*) $105\div35=3$. (*c*) $490\div35=14$, as above.

EXERCISE XXXIV.

Reduce the following to their lowest terms:

1. $\frac{296}{407}$.	2. $\frac{360}{480}$.	3. $\frac{268}{286}$.	4. $\frac{462}{770}$.
5. $\frac{105}{245}$.	6. $\frac{120}{260}$.	7. $\frac{144}{240}$.	8. $\frac{756}{1092}$.
9. $\frac{5376}{6144}$.	10. $\frac{693}{4455}$.	11. $\frac{119}{187}$.	12. $\frac{85}{323}$.
13. $\frac{288}{1728}$.	14. $\frac{406}{1595}$.	15. $\frac{591}{765}$.	16. $\frac{471}{785}$.
17. $\frac{1599}{2665}$.	18. $\frac{814}{1221}$.	19. $\frac{622}{1555}$.	20. $\frac{4234}{6351}$.
21. $\frac{322}{1127}$.	22. $\frac{2253}{3004}$.	23. $\frac{508}{889}$.	24. $\frac{609}{1421}$.
25. $\frac{492}{1107}$.	26. $\frac{518}{1221}$.	27. $\frac{2418}{2697}$.	28. $\frac{2102}{2555}$.
29. $\frac{90560}{170119}$.	30. $\frac{1032}{5635}$.	31. $\frac{336}{384}$.	32. $\frac{288}{360}$.

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Reduction of Integers to Fractional Forms.

175. Every integer may be converted into a fraction by writing 1 for its denominator.

As multiplying both terms by the same number does not alter the value of the fraction, the fraction already derived from the integer may take *any* denominator desired. Thus, $5=\frac{5}{1}=\frac{5}{1}\times\frac{7}{7}=\frac{35}{7}$. (Art. 169.)

To Reduce an Integer to a Fractional Form.

Rule.—*Write the denominator required. For the numerator take the product of the integer by the denominator.*

Exercise XXXV.

Reduce the following:

1. 48, 13, 96, 1001, 79 to twelfths;
6. 6, 9, 12, 15, 517 to nineteenths;
11. 18, 27, 33, 54, 790 to sixths;
16. 100, 420, 960, 740, 872 to tenths;
21. 7, 9, 92, 401, 730 to twentieths;
26. 17, 4, 509, 31, 72 to thousandths;
31. 5, 15, 7, 6, 4 to seventeenths.
36. Reduce 12 to 9ths; to 15ths; 13ths; 17ths; 20ths.
41. Reduce 25 to sixths; to sixteenths; seventieths.

Reduce \$23 to quarter-dollars.

Solution.—In \$1 there are 4 quarter-dollars; in \$23 there are 23 times 4 quarter-dollars, or 92 quarter-dollars.

How many melons, at 25 cents each, can be bought for \$23? How many for \$70? For \$106? For \$112?

How many pineapples, at 20 cents each, will \$5 purchase?

Solution.—1 cent is $\frac{1}{100}$ of \$1; 20 cents are $\frac{20}{100}$ of a dollar; $\$\frac{20}{100}=\$\frac{1}{5}$. In \$5 there are $\$\frac{25}{5}$, and these are 25 times the price of 1 pineapple. ∴ \$5 will purchase 25 pineapples @ 20 cents each.

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How many fifths in 12? If 12 bushels of oats grow on $\frac{1}{5}$ of an acre, how many bushels will 17 a. produce? How many will 19 a.? 27 a.? 38 a.?

If a city lot equals $\frac{1}{12}$ of an acre, how many lots will 49 a. form? How many will 316 a.?

How many hundredths in 14? How many fiftieths? How many tenths?

If a cloud travels 3 miles in one fifth of an hour, how many miles will it go in 4 hours?

Solution.—In 4 hours there are $\frac{20}{5}$ hours, and, at the rate of 3 miles in $\frac{1}{5}$ of an hour, in 20 such periods or portions of an hour, 3 miles $\times$ 20, or 60 miles, will be traversed.

Multiples and Sub-Multiples.

176. There is no limit to the number of forms which every fraction may assume. The new terms may be equi-multiples of the old; or, if both of the terms have been *divided* by the same number, then the new terms are called *equi-sub-multiples.*

Exercise XXXVI.

Write 4 new forms for each of the following:

1. $\frac{3}{4}$.	2. $\frac{7}{9}$.	3. $\frac{2}{5}$.	4. $\frac{5}{8}$.	5. $\frac{8}{7}$.
6. $\frac{7}{11}$.	7. $\frac{7}{13}$.	8. $\frac{9}{12}$.	9. $\frac{9}{13}$.	10. $\frac{11}{4}$.
11. $\frac{5}{4}$.	12. $\frac{6}{5}$.	13. $\frac{9}{15}$.	14. $\frac{18}{170}$.	15. $\frac{100}{204}$.

Write 4 sets of sub-multiples for these:

16. $\frac{32}{48}$. 17. $\frac{81}{243}$. 18. $\frac{625}{1000}$. 19. $\frac{200}{400}$. 20. $\frac{1728}{1600}$.

Reduction of Compound Fractions to Simple Ones.

177. We have seen that if the product of several numbers is to be divided by another product containing similar numbers, the work may be shortened by rejecting factors common to both dividend and divisor.

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178. The numerator of a compound fraction is a dividend made up of detached factors, and its denominator is a divisor represented by detached factors. The fraction $\frac{9}{40}$ of $\frac{8}{27}$ of $\frac{3}{5}$ of $\frac{15}{21}$ is equal to $(9\times8\times3\times15)\div40\times27\times5\times21$. The fraction might, if *cancellation* were employed, stand thus: $\frac{1}{5}$ of $\frac{1}{3}$ of $\frac{1}{1}$ of $\frac{3}{7}=\frac{1}{35}$.

179. In order to lessen the labor of multiplication and to present the result in its lowest terms, factors which appear in both terms should be promptly rejected.

To Reduce a Compound Fraction.

Rule.—*After reducing whole or mixed numbers to improper fractions, and cancelling factors common to both terms, take the product of such of the numerators as remain, for a numerator of the simple fraction, and the product of the remaining denominators for its denominator. If the result be an improper fraction, reduce it to a whole or mixed number.*

Exercise XXXVII.

Reduce the following to simple fractions (of $=\times$) :.

1. $\frac{3}{4}$ of $\frac{5}{6}$.
2. $\frac{2}{3}$ of $\frac{2}{3}$.
3. $\frac{2}{5}$ of $\frac{7}{8}$.
4. $\frac{4}{7}$ of $\frac{5}{6}$.
5. $\frac{5}{8}$ of $\frac{3}{7}$.
6. $\frac{2}{11}$ of $\frac{3}{15}$.
7. $\frac{15}{17}$ of $\frac{9}{10}$.
8. $\frac{3}{7}$ of $11\frac{2}{10}$.
9. $2\frac{1}{2}\times9\frac{1}{4}\times3\frac{1}{5}$.
10. $4\frac{1}{7}\times3\frac{1}{7}\times10\frac{1}{11}$.
11. $6\frac{1}{4}\times\frac{1}{2}$ of $4\frac{1}{7}$.
12. $8\frac{11}{12}\times\frac{2}{5}\times\frac{3}{8}$.

13. A man owns a share in a mine equal to $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of it. What part of the mine does he own?

14. A pint is $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{1}{63}$ of a hogshead. What part of a hhd. is a pint?

15. What part of a mile is an inch, if it equals $\frac{1}{12}$ of $\frac{1}{660}$ of $\frac{1}{8}$ of a mile?

16. What part of a lb. Troy is a grain?

17. What part of a gallon is a gill?

18. What part of a dollar is a mill?

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19. What part of a £ is a penny?

20. Simplify $\frac{1}{2}$ of $\frac{3}{7}$ of $\frac{21}{9}$ of $\frac{12}{14}$ of $\frac{28}{24}$;

21, 22. $\frac{2}{88}\times\frac{11}{13}\times\frac{16}{14}$; $4\frac{1}{2}\times\frac{2}{5}$ of $\frac{5}{11}$ of $3\frac{2}{3}$;

23, 24. $\frac{4}{7}$ of $\frac{9}{12}$ of $\frac{6}{4}$; $1\frac{1}{5}\times\frac{2}{3}$ of $3\frac{1}{4}$;

25, 26. $8\times\frac{18}{56}\times9\frac{2}{3}\times\frac{7}{32}$; $\frac{3}{7}\times2\times5\frac{1}{6}$.

To reduce Fractions to a Common Denominator.

180. The common multiple of the given denominators is, of course, a Common Denominator, and the L. C. M. equals the L. C. D.

181. Only *like* quantities can be added. $\frac{1}{3}$ and $\frac{1}{4}$ cannot, unless reduced, be written as one fraction. In order that they may be added, they must be reduced to the same name. $\frac{2}{3}$ and $\frac{2}{5}$ make 4 parts in all, but they are neither thirds nor fifths. Each fraction can be reduced to fifteenths, thus,

$$\frac{2\times5}{3\times5}=\frac{10}{15} \text{ and } \frac{2\times3}{5\times3}=\frac{6}{15}$$

and their sum, which is then readily found, will equal $\frac{16}{15}$.

182. To reduce Fractions to others having a Common Denominator:

Rule.—*Multiply the denominators together for the common denominator; and multiply each numerator by all the denominators except its own, for the respective numerators. Before applying the rule, compound fractions and mixed numbers must be simplified.*

183. The preceding method has sometimes the disadvantage of producing denominators inconveniently *large*. For example, the following fractions reduced by this method present the results annexed:

$$\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}=\frac{864}{1728}, \frac{1152}{1728}, \frac{432}{1728}, \frac{288}{1728}, \frac{144}{1728}.$$

Instead of this tedious process, we may find the least common multiple of the several denominators, and make it the common denominator of all the fractions; and we have $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{12}=\frac{6}{12}$, $\frac{8}{12}$, $\frac{3}{12}$, $\frac{2}{12}$, $\frac{1}{12}$.

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Since, in the example above, 12 is a multiple of the denominators severally, if we divide it by each denominator separately, the quotient multiplied by that denominator will produce the common denominator; and if the numerator be also multiplied by that quotient, the value of the fraction will remain the same.

Thus, taking the fraction $\frac{1}{2}$, and dividing the common multiple (12) by the denominator: $12 \div 2 = 6$; and $\frac{1}{2} \times \frac{6}{6} = \frac{6}{12}$, the equivalent fraction. Hence

184. To REDUCE FRACTIONS TO THE LEAST COMMON DENOMINATOR:

Find the least common multiple of the denominators for a common denominator.

Divide it by the denominator of each fraction, and multiply the several quotients by the respective numerators. Write the products over the L. C. D.

EXERCISE XXXVIII.

1. Reduce $\frac{3}{8}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$ to the least common denominator.

The least common multiple of 8, 6, 12, 3, 2, 4 = 24.

$24 \div 8 = 3$; and $3 \times 3 = 9$, the numerator of the first fraction.

$24 \div 6 = 4$; and $4 \times 5 = 20$, the numerator for $\frac{5}{6}$; and so of the rest.

Then $\frac{3}{8}, \frac{5}{6}, \frac{7}{12}, \frac{2}{3}, \frac{1}{2}, \frac{3}{4} = \frac{9}{24}, \frac{20}{24}, \frac{14}{24}, \frac{16}{24}, \frac{12}{24}, \frac{18}{24}$.

2. Reduce $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{6}$, $\frac{11}{12}$, $\frac{2}{15}$, $\frac{7}{30}$ to a common denominator.

Ans. $\frac{45}{60}, \frac{24}{60}, \frac{10}{60}, \frac{55}{60}, \frac{8}{60}, \frac{14}{60}$.

3. Reduce $1\frac{1}{3}$, and $\frac{1}{2}$ of $\frac{3}{7}$ to a common denominator.

Ans. $\frac{56}{42}, \frac{9}{42}$.

4. Reduce $3\frac{1}{2}$, $\frac{6}{7}$ of $\frac{1}{3}$, and 3 to a common denominator.

Ans. $\frac{42}{14}, \frac{4}{14}, \frac{42}{14}$.

NOTE.—Mixed and whole numbers are first reduced to improper fractions.

5. Reduce $\frac{3}{15}$, $\frac{4}{24}$, $\frac{8}{9}$, $\frac{5}{12}$ to the least common denominator.

Ans. $\frac{72}{360}, \frac{60}{360}, \frac{320}{360}, \frac{150}{360}$.

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6. Reduce $\frac{2}{5}$, $\frac{4}{6}$, $\frac{5}{9}$, $\frac{1}{3}$, and $\frac{7}{10}$ to equivalent fractions having a common denominator. *Ans.* $\frac{36}{90}$, $\frac{60}{90}$, $\frac{50}{90}$, $\frac{30}{90}$, $\frac{63}{90}$.

7. Find the least common denominator of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $12\frac{1}{3}$.

Ans. $\frac{6}{12}$, $\frac{9}{12}$, $\frac{10}{12}$, $\frac{148}{12}$.

8. Reduce $\frac{3}{7}$ of $\frac{2}{5}$ of $\frac{5}{8}$, and $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{3}{5}$ to a common denominator. *Ans.* $\frac{5}{28}$, $\frac{9}{28}$.

NOTE.—Reduce all the compound fractions to simple ones.

9. Reduce $\frac{3}{5}$, $\frac{2}{3}$, $\frac{1}{7}$, $\frac{1}{2}$ to a common denominator.

10. Reduce $2\frac{1}{3}$, $\frac{1}{2}$ of $\frac{1}{7}$, and $\frac{5}{6}$ to a common denominator.

Ans. $\frac{28}{42}$, $\frac{3}{42}$, $\frac{35}{42}$.

Reduction of Complex Fractions.

185. A **complex fraction** may be a fraction divided by a fraction, an integer divided by a fraction, or a fraction divided by an integer.

The analysis and explanation of questions in complex fractions must be referred to the section of Division of Fractions.

TO REDUCE COMPLEX FRACTIONS TO SIMPLE ONES.

RULE.—*Multiply the outside terms for a numerator and the middle terms for a denominator.*

Thus: $\frac{\frac{3}{4}}{\frac{5}{6}}=\frac{3\times6}{4\times5}=\frac{18}{20}$ $\frac{4}{\frac{3}{5}}=\frac{\frac{4}{1}}{\frac{3}{5}}$, or $\frac{20}{3}$. $\frac{\frac{4}{7}}{9}=\frac{\frac{4}{7}}{\frac{9}{1}}$, or $\frac{4}{63}$.

For problems see Ex. LII., Division of Fractions.

Section IX. Addition of Fractions.

The sum of $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, and $\frac{4}{4}$ is plainly $\frac{10}{4}$, or $2\frac{1}{4}$.

The sum of $2\frac{1}{7}$, $10\frac{3}{7}$, and $\frac{4}{7}$ is $12\frac{8}{7}$, or $13\frac{1}{7}$.

186. TO ADD FRACTIONS WHOSE DENOMINATORS ARE COMMON.

RULE.—*Add the numerators, and write the sum over the common denominator. If there be integers, they form, at first, a separate sum. The result should appear in its lowest terms.*

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EXERCISE XXXIX.

1. $\frac{5}{7}+1\frac{2}{7}+2\frac{3}{7}+3\frac{4}{7}=?$
2. $\frac{3}{5}+10\frac{2}{5}+11\frac{1}{5}+4\frac{3}{5}=?$
3. $2\frac{1}{2}+3\frac{1}{2}+\frac{1}{2}+\frac{5}{2}=?$
4. $\frac{3}{5}$ of $\frac{2}{3}+\frac{1}{3}$ of $\frac{2}{5}=?$
5. $5\frac{5}{9}+\frac{4}{9}+7\frac{1}{9}=?$
6. $6\frac{1}{10}+4\frac{3}{10}+7\frac{2}{10}=?$
7. $5\frac{1}{8}+3\frac{5}{8}+5\frac{3}{8}=?$
8. $7\frac{1}{3}+2\frac{1}{2}+4\frac{1}{2}=?$

NOTE.—The teacher should dictate similar problems.

187. To add Dissimilar Fractions.

RULE.—*If there be any compound or complex fractions, reduce them to simple ones; then reduce all to a common denominator, and add the numerators. Place the result over the common denominator, and reduce this result, if possible, to a whole or mixed number. Integers should be added separately.*

188. In reducing the fractions to their L. C. D., find the L. C. M. of the denominators for the L. C. D., and divide it by each denominator; multiply each numerator by the respective quotient.

1. Required the sum of $10\frac{1}{5}$, $3\frac{3}{4}$, $\frac{5}{6}$, $7\frac{2}{15}$, and 2.

$$\frac{1}{5}+\frac{3}{4}+\frac{5}{6}+\frac{2}{15}=\frac{12+45+50+8}{60}=1\frac{15}{60}=1\frac{5}{60}=1\frac{1}{12}.$$

$$10+3+7+2=22;\ 22+1\frac{1}{12}=23\frac{1}{12}.$$

189. Errors will be avoided and explanations simplified if, as a question is *written*, parentheses are placed around compound fractions.

Example 2. $4\frac{5}{7}+2\frac{1}{12}$ of $1\frac{1}{5}+6\frac{1}{4}$ of $2\frac{7}{10}$ of $\frac{1}{9}=?$

$4\frac{5}{7}+(2\frac{1}{12}$ of $1\frac{1}{5})+(6\frac{1}{4}$ of $2\frac{7}{10}$ of $\frac{1}{9})=$

$4\frac{5}{7}+(\frac{25}{12}\times\frac{6}{5})+(\frac{25}{4}\times\frac{27}{10}\times\frac{1}{9})$.

As $\frac{\overset{5}{\cancel{25}}}{\underset{2}{\cancel{12}}}\times\frac{\overset{1}{\cancel{6}}}{\underset{1}{\cancel{5}}}=2\frac{1}{2}$, and $\frac{\overset{5}{\cancel{25}}}{\underset{1}{\cancel{4}}}\times\frac{\overset{3}{\cancel{27}}}{\underset{2}{\cancel{10}}}\times\frac{1}{\underset{1}{\cancel{9}}}=\frac{15}{8}=1\frac{7}{8}$. Adding the fractions $(\frac{5}{7}+\frac{1}{2}+\frac{7}{8})$, or $\frac{40+28+49}{56}=1\frac{17}{56}=2\frac{5}{56}$; which, added to the integers $=4+2+1+2+\frac{5}{56}=9\frac{5}{56}$, the answer.

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190. *Analysis.*—(*a*) Mark off each complete fraction.
(*b*) Simplify those which are compound.
(*c*) Reduce the simple fractions to L.C.D.
(*d*) Add the simple fractions, and reduce the sum.
(*e*) Add the sum of the integers to the proper fraction.

Exercise XL.

1. $\frac{9}{10}+\frac{3}{5}+\frac{5}{20}+\frac{2}{15}$.
2. $\frac{1}{4}$ of $\frac{3}{7}+\frac{2}{9}+1\frac{1}{2}$.
3. $\frac{3}{4}+\frac{5}{6}+4\frac{1}{4}+\frac{1}{3}$.
4. $\frac{2}{13}+\frac{5}{12}+\frac{7}{15}+5$.
5. $\frac{5}{11}+\frac{17}{22}+\frac{3}{8}+6$.
6. $\frac{1}{2}$ of $\frac{1}{20}$ of $\frac{5}{8}+3$.
7. $\frac{3}{7}$ of $\frac{9}{16}$ of $\frac{15}{28}+4$.
8. $\frac{1}{5}$ of $\frac{3}{10}$ of $\frac{7}{18}+2$.
9. $3\frac{1}{5}+2\frac{3}{4}+5\frac{11}{15}$.
10. $1\frac{5}{19}+2\frac{6}{7}+\frac{11}{38}+6$

191. If **two** Fractions have **units** for Numerators.

Rule.—*Write the sum of the denominators over their product.*

Exercise XLI.

1–18. $\frac{1}{7}+\frac{1}{13}$; $\frac{1}{9}+\frac{1}{10}$; $\frac{1}{4}+\frac{1}{12}$; $\frac{1}{3}+\frac{1}{7}$; $\frac{1}{4}+\frac{1}{11}$; $\frac{1}{5}+\frac{1}{9}$; $\frac{1}{15}+\frac{1}{2}$; $\frac{1}{16}+\frac{1}{3}$; $\frac{1}{10}+\frac{1}{20}$; $\frac{1}{5}+\frac{1}{100}$; $\frac{1}{12}+\frac{1}{6}$; $\frac{1}{90}+\frac{1}{19}$; $\frac{1}{7}+\frac{1}{70}$; $\frac{1}{4}+\frac{1}{40}$; $\frac{1}{9}+\frac{1}{900}$; $\frac{1}{12}+\frac{1}{1200}$; $\frac{1}{5}+\frac{1}{19}$; $\frac{1}{7}+\frac{1}{23}$.

Note.—Dictation exercises in order.

192. To add Fractions which have common Numerators.

Rule.—*Take the product of the denominators for the new denominator, and multiply the common numerator into the sum of the denominators for the new numerator.*

Exercise XLII.

1–12. $\frac{2}{5}+\frac{2}{7}$; $\frac{2}{3}+\frac{2}{4}$; $\frac{7}{9}+\frac{7}{11}$; $\frac{5}{6}+\frac{5}{16}$; $\frac{8}{9}+\frac{8}{12}$; $\frac{9}{11}+\frac{9}{2}$; $\frac{3}{4}+\frac{3}{15}$; $3\frac{1}{5}+4\frac{1}{8}$; $5\frac{6}{11}+7\frac{6}{8}$; $9\frac{1}{10}+8\frac{1}{12}$; $5\frac{7}{11}+6\frac{7}{9}$; $5\frac{3}{12}+3\frac{3}{15}$.

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Section X. Subtraction of Fractions.

193. ORAL LESSON.—From 6 pins take 4 pins. From 12 figs take 7 figs. From two halves take one half. From $\frac{3}{4}$ take $\frac{1}{4}$. From 10 equal parts of anything take away 9 parts. Suppose the parts to be tenths, what will remain? Suppose them to be 12ths? 20ths? 100ths? What remains if from $\frac{9}{10}$ $\frac{4}{10}$ be taken? What if $\frac{6}{10}$? $\frac{7}{10}$? $\frac{3}{10}$? What is the difference between $\frac{5}{7}$ and $\frac{2}{7}$? What are the following expressions equal to, viz.: $\frac{3}{5}-\frac{2}{5}$; $\frac{8}{7}-\frac{5}{7}$; $\frac{9}{10}-\frac{6}{10}$; $\frac{19}{2}-\frac{17}{2}$?

194. TO FIND THE DIFFERENCE BETWEEN TWO FRACTIONS.

RULE.—*Reduce the fractions as in Addition of Fractions, and find the difference between the numerators and write it over the common denominator.*

195. *If mixed numbers occur, annex the difference of the fractions to the difference of the integers.*

196. *Example* 1. $4\frac{3}{7}-2\frac{2}{7}=2\frac{1}{7}$.

Example 2. $4\frac{2}{3}-2\frac{2}{7}=(3-2)+(1\frac{2}{3}-\frac{2}{7})=1\frac{14}{63}$; for $3-2=1$, and $1\frac{2}{3}=1\frac{14}{21}$ or $\frac{77}{63}$ and $\frac{2}{7}=\frac{18}{63}$; $\therefore 1\frac{2}{3}-\frac{2}{7}=\frac{59}{63}$.

EXERCISE XLIII.

Find the difference between the following:

1–5. $\frac{4}{15}$ and $\frac{2}{15}$; $1\frac{2}{17}$ and $\frac{9}{17}$; $\frac{3}{12}$ and $\frac{1}{12}$; $\frac{52}{2}$ and $\frac{50}{2}$; $\frac{16}{18}$ and $\frac{10}{18}$.

6–10. $\frac{1}{2}$ and $\frac{1}{3}$; $\frac{1}{6}$ and $\frac{1}{4}$; $\frac{2}{5}$ and $\frac{2}{6}$; $\frac{2}{5}$ and $\frac{2}{7}$; $5\frac{3}{8}$ and $2\frac{3}{8}$.

What numbers equal the following expressions:

11–15. $\frac{4}{5}-\frac{3}{7}$; $1\frac{3}{4}-\frac{7}{9}$; $16\frac{1}{3}-14\frac{11}{12}$; $143\frac{1}{9}-\frac{5}{6}$; $210\frac{3}{5}-169\frac{1}{7}$.

16–20. $148\frac{1}{9}-130\frac{10}{11}$; $\frac{2}{5}-\frac{2}{6}$; $\frac{7}{12}-\frac{8}{15}$; $30\frac{1}{4}-20\frac{1}{3}$; $21\frac{1}{2}-4\frac{1}{7}$.

21–25. $\frac{3}{5}$ of $\frac{2}{7}-\frac{3}{40}$; $\frac{10}{11}-\frac{2}{3}$ of $\frac{1}{7}$; $4\frac{1}{2}-\frac{1}{2}$ of $\frac{1}{2}$; $\frac{12}{13}-\frac{12}{14}$; $\frac{5}{6}-\frac{3}{11}$?

26–30. What must be added to $\frac{1}{5}$ to make $2\frac{1}{3}$; to make $3\frac{1}{4}$; $4\frac{1}{7}$; $5\frac{1}{6}$; $3\frac{1}{10}$?

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31–35. What must be added to $2\frac{1}{11}$ to make $3\frac{1}{7}$; to make $4\frac{1}{2}$; $5\frac{2}{5}$; 12; $13\frac{1}{5}$?

36–40. What must be added to $\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{4}{7}$ to make $2\frac{1}{4}$; to make $3\frac{1}{7}$; $2\frac{10}{13}$; $\frac{2}{3}$ of $\frac{7}{10}$; $3\frac{1}{2}$?

197. When both Numerators are Units.

Rule.—*Write the difference of the denominators over the product of the denominators—e. g.,* $\frac{1}{3}-\frac{1}{9}=\frac{4}{45}$; $\frac{1}{3}-\frac{1}{11}=\frac{8}{33}$.

Note to Teacher.—Dictate several short exercises besides the following.

Exercise XLIV.

1–14. $\frac{1}{3}-\frac{1}{6}$; $\frac{1}{10}-\frac{1}{11}$; $\frac{1}{13}-\frac{1}{17}$; $\frac{1}{4}-\frac{1}{7}$; $\frac{1}{7}-\frac{1}{9}$; $\frac{1}{3}-\frac{1}{5}$; $\frac{1}{4}-\frac{1}{10}$; $\frac{1}{2}-\frac{1}{5}$; $\frac{1}{5}-\frac{1}{10}$; $\frac{1}{20}-\frac{1}{30}$; $\frac{1}{10}-\frac{1}{100}$; $\frac{1}{4}-\frac{1}{13}$; $\frac{1}{3}-\frac{1}{15}$; $\frac{1}{7}-\frac{1}{10}$.

198. When the Numerators are Alike.

Rule.—*Write the difference of the denominators, multiplied by the common numerator, over the product of the denominators—e. g.,* $\frac{7}{5}-\frac{7}{8}=\frac{3\times 7}{5\times 8}$, or $\frac{21}{40}$.

Note.—Explain carefully that this result, in every instance, is equal to that obtained by the common process.

Exercise XLV.

1–10. $\frac{3}{4}-\frac{3}{5}$; $\frac{12}{5}-\frac{12}{10}$; $\frac{13}{2}-\frac{13}{7}$; $\frac{12}{6}-\frac{12}{14}$; $\frac{7}{8}-\frac{7}{15}$; $\frac{17}{42}-\frac{17}{52}$; $\frac{14}{108}-\frac{14}{117}$; $\frac{1006}{4223}-\frac{1006}{13564}$; $4\frac{7}{12}-2\frac{7}{16}$; $21\frac{12}{108}-1\frac{12}{104}$; $\frac{35}{18}-\frac{35}{16}$.

199. Use of Parentheses.

It must be remembered that every quantity in a parenthesis must be subjected to the **same** operation; also that the effect of any multiplier or divisor never extends beyond the sign plus or the sign minus, unless the quantities which it connects are within a parenthesis or under a vinculum.

Example 1. $(5+9-\frac{1}{3})\times 4=(5\times 4)+(9\times 4)-(\frac{1}{3}\times 4)=13\frac{2}{3}\times 4$.

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Example 2. $(\frac{1}{2}+\frac{2}{7})-(\frac{3}{5}+\frac{1}{3})\times\frac{4}{5}=\frac{11}{14}-\frac{34}{45}\times\frac{4}{5}=\frac{11}{14}-\frac{136}{225}$. If the last parenthesis in the example were not regarded, then the process would be as follows: $(\frac{1}{2}+\frac{2}{7})-\frac{3}{5}+\frac{1}{3}\times\frac{4}{5}=\frac{11}{14}-\frac{3}{5}+\frac{4}{45}=\frac{11}{14}-\frac{23}{45}$.

200. A line drawn over or under several numbers is called **a vinculum.** *It is equivalent to a parenthesis.*

Miscellaneous Questions.

Exercise XLVI.

(1.) $(12-3)\div3$; (2.) $(24-8)\times4$; (3.) $(45+55)\div10$.

(4.) $\frac{84-9+15}{15}$; (5.) $\frac{2\frac{1}{2}+3\frac{1}{2}}{8\frac{1}{4}+3\frac{3}{4}}$; (6.) $\frac{(49\times7-43)\div30}{10}$.

(7.) $1006\frac{2}{5}-75\frac{1}{9}-148\frac{4}{7}-548\frac{1}{11}+426\frac{1}{4}-648\frac{1}{2}=?$

(8.) A man had $\$1242\frac{1}{6}$. He first expended $\$175\frac{1}{10}$, then $\$271\frac{1}{9}$; he then received $\$42\frac{1}{12}$; he then spent $\$548\frac{1}{11}$; and laid out $\$416\frac{2}{19}$. What had he remaining?

(9.) How much is required to make the difference between $2\frac{3}{7}$ and $1\frac{3}{4\,2}$ equal to the sum of $\frac{1}{14}$, $1\frac{5}{8}$, and $\frac{3}{51}$?

(10.) By how much does the sum of $30\frac{5}{8}$ and $6\frac{2}{7}$ exceed the sum of $10\frac{3}{11}$ and $7\frac{2}{15}$?

(11.) A man owns $\frac{5}{16}$ of a ship worth $\$42,654$, $\frac{3}{32}$ of a mine estimated as worth $\$500,000$, $\frac{1}{12}$ of a mill worth $\$34,000$, and has a half interest in a farm worth $\$12,000$. What is his fortune?

(12.) What will make the difference between $2\frac{4}{7}$ and $1\frac{3}{4\,2}$ equal the sum of $\frac{1}{9}$, $1\frac{4}{7}$, and $3\frac{1}{4}$?

(13.) Erie R.R. stock is quoted at $34\frac{1}{8}$, Pennsylvania R.R. stock at $91\frac{1}{4}$, Hudson River R.R. at $102\frac{1}{4}$, New Haven R.R. at $139\frac{1}{4}$. What is the entire value of 5 shares of each?

(14.) If Mich. Central R.R. stock is worth $75\frac{1}{4}$, Rock Island $103\frac{1}{4}$, Union Pacific $75\frac{1}{8}$, Pittsburgh and Fort Wayne $92\frac{1}{8}$, and Lake Shore $73\frac{1}{8}$, what is the sum of the values of 1 share of each?

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Section XI. Multiplication of Fractions.

201. There may be required:

I. The product of an integer and a simple fraction.

II. The product of an integer and a mixed number.

III. The product of two mixed numbers.

IV. The product of two fractions.

202. I. To Multiply a Fraction by an Integer, or an Integer by a Fraction.

Rule 1. *Multiply the numerator of the fraction by the integer; or,*

2. *Divide the denominator of the fraction by the integer.*

Thus, in the question $3\times\frac{7}{12}$, or $\frac{7}{12}$ of 3, we may have $\frac{21}{12}$, or $\frac{7}{4}$, each equal to $1\frac{3}{4}$. The second method is to be preferred when the integer is exactly contained in the denominator.

Note.—In a compound fraction, **of** is equivalent to the sign $\times$.

Exercise XLVII.

1–12. $4\times\frac{2}{12}$; $\frac{1}{7}$ of 16; $\frac{4}{5}$ of 25; $\frac{3}{5}\times 10$; $3\times\frac{5}{9}$; $\frac{2}{9}$ of 3; $\frac{5}{6}$ of 186; $\frac{5}{7}\times 4$; $7\times\frac{2}{21}$; $\frac{3}{5}$ of 20; $\frac{2}{9}$ of 279; $\frac{3}{8}\times 7$.

203. II. When one Factor is a Mixed Number.

Rule.—*First multiply the integer, then multiply the fraction. Finally add the two products.*

1, 2. $4\frac{1}{9}\times 7=28+\frac{7}{9}$; $7\times 4\frac{5}{6}=28+\frac{35}{6}=33\frac{5}{6}$.

3–6. $73\frac{5}{6}\times 12$; $1611\frac{5}{13}\times 5$; $291\frac{5}{7}\times 7$; $47\times 23\frac{1}{4}$.

7. If one sheep cost $5\frac{1}{8}$ dollars, what cost 14 sheep?

8. If one swing cost $4\frac{1}{3}$ dollars, what cost 7 swings?

9. If one hat cost $4\frac{1}{2}$ dollars, what will 4 times 7 hats cost?

10. If one plough is worth $3\frac{5}{8}$ dollars, what are 9 ploughs worth?

11. If one hoe is worth $62\frac{1}{2}$ cents, what will 9 hoes cost?

12. If one harrow is worth $\$14\frac{2}{5}$, what will 3 harrows cost?

13. If a man buys a chain at $\$17\frac{3}{8}$ an ounce, what must he pay if it weigh 5 ounces?

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14. When hay sells at $\$19\frac{5}{12}$ per ton, what are 22 tons worth?

15. If one house cost $\$2416\frac{7}{8}$, what will a row of 6 houses cost?

16. What cost 8 bearskins at $\$21\frac{3}{8}$ each?

EXERCISE XLVIII.

1–32. $3\frac{7}{9}$ of $81+20$; $\frac{3}{5}$ of $4+\frac{2}{3}$ of 8; $\frac{3}{4}$ of $9+\frac{3}{7}$;
$\frac{1}{7}$ of $9+\frac{2}{3}$ of 12; $\frac{5}{6}$ of $24+19$; $\frac{5}{2}$ of $16+\frac{1}{9}$ of 18;
$\frac{3}{5}$ of $105+7\frac{1}{2}$; $7\times2\frac{1}{2}+14\frac{1}{5}$; $5\frac{1}{6}\times4-3\frac{1}{3}$;
$3\times4+5\times9\frac{1}{10}$; $72\frac{6}{7}\times12-1$; $2\frac{1}{2}\times4-\frac{1}{3}$ of 12;
$8\frac{1}{2}\times4-2\frac{1}{4}$; $3\frac{1}{5}+\frac{4}{5}$ of 21; $\frac{4}{5}$ of $7640+8$;
$2\frac{1}{4}+\frac{1}{2}$ of 7; $41\frac{3}{8}\times16+4$; $21\frac{1}{6}\times4+9$;
$5\frac{1}{2}\times(2+3)$; $3\frac{1}{8}\times24+2\frac{1}{4}$; $(4+2\frac{1}{7})\times6$;
$3\frac{1}{10}\times5+106\frac{1}{4}$; $(\frac{3}{5}-\frac{1}{7})\times3$; $8\times7\frac{1}{2}+15$;
$\frac{6}{7}\times8+\frac{4}{5}$; $135+4\times3\frac{1}{5}$; $\frac{3}{8}$ of $7+\frac{9}{10}$;
$22\frac{1}{4}\times9+161$; $\frac{2}{7}$ of $14+1\frac{1}{5}$; $28\frac{1}{5}-\frac{3}{4}$ of 30;
$\frac{3}{4}$ of $24+2\frac{1}{7}$; $19\frac{1}{5}-\frac{1}{5}$ of 60.

204. III. To FIND THE PRODUCT OF TWO FRACTIONS.

RULE.—*Take the product of the numerators for the numerator, and the product of the denominators for the denominator.*

Let it be required to find the product of $\frac{4}{5}$ and $\frac{2}{3}$.

First Analysis.—$\frac{1}{3}$ of $\frac{4}{5}=\frac{4}{15}$; $\frac{2}{3}$ of $\frac{4}{5}$ is twice as much, or $\frac{8}{15}$.

Second Analysis.—Twice $\frac{4}{5}$ is $\frac{8}{5}$. The multiplier is not really 2; it is $\frac{1}{3}$ of 2. Therefore $\frac{8}{5}$ is too large; for the multiplier employed was 3 times too great. $\therefore\frac{1}{3}$ of $\frac{8}{5}$, or $\frac{8}{15}$, is the correct result; for fifteenths are thirds of fifths.

To multiply by $\frac{2}{3}$ is to take $\frac{1}{3}$ of twice the multiplicand; to multiply by $\frac{7}{11}$ is to take $\frac{1}{11}$ of 7 times the multiplicand.

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205. To multiply by a fraction is to multiply by the numerator and to take that part of the product which is indicated by the denominator. In every fraction the denominator is a divisor.

Exercise XLIX.

Required the simplest equivalents of the following:

1–34. $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{2}{3}$. $\frac{9}{11}$ of $7\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{1}{6}$. $\frac{4}{5}$ of $\frac{7}{8}$ of $1\frac{2}{3}$.

$\frac{1}{9}$ of $2\frac{1}{2}\times\frac{6}{20}$ of 8. $\frac{2}{5}$ of $\frac{1}{3}$ of 6. $5\times\frac{12}{21}\times4\frac{1}{6}$.

$2\frac{1}{2}\times1\frac{9}{15}\times\frac{3}{18}$. $\frac{18}{5}$ of $\frac{20}{12}$ of $\frac{1}{24}$ of $\frac{140}{630}$. $\frac{2}{3}$ of $2\frac{1}{2}\times3$.

$3\frac{3}{4}$ cords @ $\$1\frac{7}{8}$. $\frac{4}{7}$ of $\frac{7}{8}$ of 32. $6\frac{4}{7}\times4\frac{9}{10}$.

$1\frac{1}{9}\times4\frac{8}{12}\times\frac{10}{7}\frac{8}{}$. $7\frac{3}{5}\times6\frac{7}{12}$. $9\frac{1}{6}\times2\frac{3}{8}\times4$. $6\times\frac{2}{3}$ of 5.

$\frac{1}{9}$ of $\frac{1}{10}$ of 90. $\frac{3}{4}$ of $6\times\frac{2}{3}$ of $\frac{1}{3}$. $\frac{7}{12}$ of $\frac{24}{21}$ of 9.

$5\times\frac{4}{21}\times\frac{15}{60}$. $3\frac{1}{5}\times4\frac{1}{2}\times12$. $\frac{2}{15}$ of $3\frac{1}{7}\times\frac{5}{8}$. $\frac{4}{11}$ of $\frac{3}{7}$ of $\frac{28}{3}$.

$\frac{11}{12}$ of $\frac{8}{9}$ of $\frac{6}{21}$ of $\frac{3}{4}$. $11\frac{1}{2}\times1205\frac{1}{3}$. $1\frac{1}{2}\times1\frac{1}{3}\times4\frac{1}{4}$.

$\frac{404}{5}\times\frac{100}{202}\times3$. $1\frac{3}{4}\times2\frac{1}{2}\times2\frac{1}{7}$. $18\frac{1}{3}\times\frac{1}{7}\times30$. $\frac{4}{7}\times\frac{5}{9}\times3\frac{1}{5}$.

$491\frac{1}{3}\times\frac{65}{3}\times\frac{1}{7}$. $8\times3\frac{1}{3}\times7\frac{1}{4}$. $3\frac{1}{4}\times2\frac{1}{2}\times\frac{1}{2}$ of 6. $4\frac{1}{2}\times4\frac{2}{7}$.

35. What cost $14\frac{1}{7}$ lbs. of silk at $\$\frac{7}{8}$ per oz.?

36. What cost $3\frac{1}{5}$ oz. of silk at $\$\frac{5}{8}$ per oz.?

37. What cost $7\frac{1}{2}$ lbs. butter at $\$\frac{3}{8}$ per lb.?

38. How far can a man walk in $12\frac{1}{5}$ hours, at the rate of $3\frac{1}{4}$ miles an hour?

39. Required the cost of $16\frac{1}{7}$ lbs. beef at $18\frac{3}{4}$ cents a lb.?

40. If railroad fare is $2\frac{3}{4}$ cents a mile, what must be paid for a journey of $426\frac{1}{2}$ miles?

41. If a merchant gain $\$45\frac{1}{10}$ by selling a case of goods for $\$95\frac{1}{3}$, what was the cost of $\frac{2}{5}$ of the goods?

42. The sum of a certain fraction added to $\frac{1}{4}$ of $16\frac{1}{9}$, is $\frac{3}{5}+24\frac{1}{7}$; what is the fraction?

43. If a cow cost $\$45\frac{1}{7}$; a sheep, $\$5\frac{2}{5}$; a turkey, $\$1\frac{7}{10}$; a hen, $\$\frac{5}{4}$; what will 2 cows, 3 sheep, 4 turkeys, and 5 hens cost?

44. A man spent at one store $\$14\frac{1}{8}$, at another store $\frac{1}{2}$ of $\frac{1}{3}$

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as much; at another, $3\frac{1}{4}$ as much as at the second store; how much did he have left of a $50 note?

45. Frank has 32 cents, John has $2\frac{1}{8}$ times as much lacking $\frac{1}{2}$ of 16 cents, and Harry has $\frac{1}{2}$ of $\frac{1}{3}$ as many as John; what has Harry?

Section XII. Division of Fractions.

206. Questions under this head may be classified as follows:

I. The division of a fraction by an integer.

II. The division of an integer by a fraction.

III. The division of a fraction by a fraction.

Let it be required to divide $\frac{8}{7}$ by 4.

To divide by 4 is to take one fourth; one fourth of 8 parts is 2 parts; and as these parts are sevenths, $\frac{1}{4}$ of $\frac{8}{7}$ is $\frac{2}{7}$.

Again, as a fraction whose parts are one fourth as large as those of another, the numerators being equal, is just one fourth of that other; so $\frac{1}{4}$ of $\frac{8}{7}$ is $\frac{8}{28}$, for 28ths are but one fourth as large as sevenths. It appears in this case that the denominator is multiplied, and that the fraction is thus divided. This will also appear from the principles of division; for, if the divisor is multiplied, the quotient is divided: the denominator *is* the divisor, and the quotient is the value of the fraction.

207. I. To divide a Fraction by an Integer.

Rule.—*Either divide the numerator by the integer, or multiply the denominator by it.*

Exercise L.

Find the quotients of the following expressions:

1–42. $\frac{8}{7} \div 2$; $\frac{15}{16} \div 5$; $\frac{15}{16} \div 4$; $\frac{8}{9} \div 3$; $\frac{14}{11} \div 7$; $\frac{21}{42} \div 9$; $\frac{21}{25} \div 7$; $\frac{13}{25} \div 26$; $4261\frac{2}{3} \div 3$; $41\frac{1}{2} \div 4$; $\frac{7}{12} \div 7$; $305\frac{1}{6} \div 12$; $141\frac{1}{6} \div 11$;

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$\frac{21}{28} \div 9$; $\frac{5}{12} \div 8$; $\frac{7}{15} \div 14$; $\frac{9}{24} \div 6$; $\frac{7}{18} \div 15$; $\frac{9}{13} \div 2$; $2\frac{1}{7} \div 3$; $2\frac{1}{14} \div 2$; $12\frac{1}{3} \div 3$; $4\frac{1}{5} \div 4$; $3\frac{6}{10} \div 5$; $2\frac{5}{17} \div 6$; $3\frac{1}{7} \div 7$; $4\frac{1}{11} \div 8$; $14\frac{2}{35} \div 9$; $16\frac{3}{8} \div 10$; $5\frac{4}{9} \div 11$; $13\frac{2}{5} \div 12$; $17\frac{3}{7} \div 13$; $14\frac{1}{9} \div 14$; $15\frac{2}{7} \div 15$; $16\frac{3}{17} \div 16$; $19\frac{1}{2} \div 17$; $21\frac{1}{3} \div 18$; $23\frac{1}{7} \div 19$; $18\frac{1}{7} \div 20$; $36\frac{2}{5} \div 21$; $43\frac{1}{11} \div 22$.

208. II. To divide an Integer by a Fraction.

Let it be required to divide 6 by $\frac{3}{5}$.

First Analysis.—$6 \div 3 = 2$; $6 \div \frac{1}{5}$ of 3 should be 5 times 2 or 10, for $\frac{1}{5}$ as large a divisor should give 5 times as great a quotient.

Second Analysis.—$6 \div 1 = 6$; $6 \div \frac{1}{5}$ of $1 = 30$, then $6 \div \frac{3}{5}$ should equal $\frac{1}{3}$ of 30 or 10. (*See the Principles of Division.*)

From each of these analyses we learn that the numerator of the divisor is used as a divisor, and the denominator as a multiplier. This application of its terms is the opposite of their use in Multiplication.

209. Note.—As the result obtained by dividing first and multiplying afterward is the same as that obtained by first using the multiplier, and then employing the divisor, either method may be adopted, *e. g.*, $(6 \div 3) \times 5 = (6 \times 5) \div 3$.

Rule.—*Divide by the numerator and multiply by the denominator.*

Exercise LI.

Find the quotients which equal the following:

1–24. $4 \div \frac{3}{11}$; $5 \div \frac{2}{7}$; $7 \div \frac{3}{5}$; $9 \div \frac{2}{7}$; $10 \div \frac{3}{15}$; $11 \div 5\frac{1}{2}$; $12 \div \frac{1}{2}$ of $\frac{2}{7}$; $13 \div \frac{2}{7}$ of $\frac{5}{9}$; $5 \div \frac{2}{13}$; $6 \div \frac{1}{9}$ of $\frac{2}{5}$; $7 \div 3\frac{3}{4}$; $8 \div 5\frac{1}{6}$; $9 \div 6\frac{1}{7}$; $8 \div 4\frac{2}{11}$; $3 \div 5\frac{1}{7}$; $7 \div 3\frac{1}{5}$; $7 \div (\frac{1}{2} + \frac{1}{5})$; $9 \div (\frac{2}{3} + \frac{1}{4})$; $12 \div (\frac{1}{7} + \frac{5}{9})$; $13 \div (\frac{2}{6} + \frac{3}{7})$; $12 \div (\frac{3}{8} + \frac{5}{11})$; $6 \div (\frac{4}{5} + \frac{1}{5})$; $3 \div 1\frac{1}{2} + \frac{5}{8}$; $4 \div (2\frac{1}{5} + 3\frac{1}{4})$.

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210. NOTE.—Teachers will have observed that the term "Integer" has been employed for the phrase "Whole number." While we would by no means advise the use of technical terms instead of common language in ordinary conversation, yet *convenient technical terms should be carefully explained and unhesitatingly employed.* Every craft requires its own peculiar tools, and each craftsman must get familiar with his implements. Such terms as factor, measure, L. C. D., L. C. M., per cent., area, square, cube, angle, and parallelopiped are as useful, and as necessary to clear conceptions, as sum, quotient, or remainder. The rule above might read, "Multiply the integer by the reciprocal of the divisor." Let us explain *reciprocal.* Its synonym, we will say, is alternate or opposite. 1 multiplied by 5 is the reciprocal of 1 divided by 5. 1 multiplied by 3 and divided by $4=\frac{3}{4}$; 1 multiplied by 4 and divided by $3=\frac{4}{3}$; $\frac{3}{4}$ is the reciprocal of $\frac{4}{3}$. If we invert a fraction we have its reciprocal. Again, $1\times 20=\frac{20}{1}$; $1\div 20=\frac{1}{20}$; so $1\times\frac{3}{4}=1\div\frac{4}{3}$. *A reciprocal of any number is equal to* 1 *divided by the number.*

211. III. TO DIVIDE A FRACTION BY A FRACTION.

Example (*a*). Divide $\frac{2}{5}$ by $\frac{3}{4}$.

$\frac{2}{5}\div 3$ would equal, as we have seen (I.), $\frac{2}{15}$.

$\frac{2}{5}\div 1$ fourth of 3 will give 4 times as large a quotient, or $\frac{8}{15}$; *i.e.*, $\frac{2}{5}\times\frac{4}{3}$.

Again $\frac{2}{5}$ and $\frac{3}{4}$, if reduced to a common denominator, equal $\frac{8}{20}$ and $\frac{15}{20}$. 8 parts will contain 15 equal parts $\frac{8}{15}$ times, or $\frac{2}{5}\times\frac{4}{3}$. On the basis of these two analyses we may formulate the following:

RULE.—*Invert the divisor and multiply the numerators for the quotient-numerator, and multiply the denominators for the quotient-denominator.*

Fractions that have a common denominator are to each other as their numerators; that is, in comparing

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such fractions, that which has a numerator twice as large as another, is twice as great a fraction. Four tenths contain two tenths just as often as 4 cents contain 2 cents, or as 4 pints contain 2 pints. By this reasoning we conclude that division by a fraction may be clearly explained by reducing both dividend and divisor to a common denominator, and dividing the numerator of the dividend by the numerator of the divisor.

212. NOTE.—This rule might be restated thus: *Reduce the given fractions to a common denominator, and then divide the numerator of the dividend by the numerator of the divisor.* It would be well to have the pupils solve some 20 questions after this formula, and that they should also perform each question in accordance with the briefer rule first given. Teachers might also present this analysis: Req. to divide $\frac{2}{5}$ by $\frac{3}{4}$. $\frac{2}{5} \div 1 = \frac{2}{5}$; $\frac{2}{5} \div \frac{1}{4}$ of $1 = 4 \times \frac{2}{5}$ or $\frac{8}{5}$, and $\frac{2}{5} \div$ by 3 fourths of 1 would equal $\frac{1}{3}$ of $\frac{8}{5}$ or $\frac{8}{15}$. (*See principles of Division.*)

EXERCISE LII.

Find the quotients of the following:

1–38. $\frac{1}{2} \div \frac{1}{2}$; $\frac{1}{2} \div \frac{1}{4}$; $\frac{1}{2} \div \frac{1}{3}$; $\frac{1}{2} \div \frac{1}{7}$; $\frac{2}{3} \div \frac{1}{3}$; $\frac{2}{3} \div \frac{1}{4}$; $\frac{2}{3} \div \frac{3}{4}$; $\frac{2}{3} \div \frac{2}{5}$; $\frac{3}{4} \div \frac{2}{3}$; $\frac{3}{4} \div \frac{5}{6}$; $\frac{1}{11} \div \frac{1}{12}$; $\frac{1}{12} \div \frac{1}{13}$; $\frac{1}{7} \div \frac{1}{10}$; $\frac{4}{6} \div \frac{3}{5}$; $\frac{3}{5} \div \frac{4}{6}$; $\frac{7}{8} \div \frac{9}{10}$; $\frac{9}{10} \div \frac{7}{8}$; $\frac{6}{7} \div \frac{6}{10}$; $\frac{6}{10} \div \frac{6}{7}$; $\frac{5}{7} \div \frac{5}{9}$; $\frac{5}{12} \div \frac{5}{15}$; $\frac{3}{5} \div \frac{3}{8}$; $\frac{6}{9} \div \frac{6}{7}$; $\frac{8}{13} \div \frac{8}{17}$;

$\frac{\frac{1}{2}}{\frac{2}{7}}$; $\frac{\frac{3}{7}}{\frac{4}{10}}$; $\frac{\frac{5}{8}}{\frac{6}{11}}$; $\frac{\frac{3}{4}}{\frac{7}{10}}$; $\frac{\frac{8}{14}}{\frac{9}{11}}$; $\frac{\frac{1}{2}}{\frac{2}{3} \text{ of } \frac{4}{5}}$; $\frac{\frac{1}{7}}{2}$; $\frac{\frac{2}{3}}{5}$; $\frac{\frac{6}{7}}{6}$; $\frac{\frac{7}{10}}{8}$; $\frac{\frac{3}{4}}{9}$; $\frac{\frac{2}{7}}{10}$;

$\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{6}{7}$ of $\frac{8}{9} \div \frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{11}$;

$2\frac{1}{2} \times 3\frac{3}{4} \times \frac{2}{7} \times \frac{8}{11} \div \frac{7}{12} \times \frac{11}{6} \times \frac{4}{18} \times \frac{6}{14}$.

213. QUESTIONS TO FOLLOW FRACTIONS.

Define the words integer, fraction, numerator, denominator, terms, simple, compound, proper, improper, mixed number, and fractional unit. What change may a mixed number

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undergo? How may an integer be reduced to a fraction? What is meant by the reduction of a fraction? How may a fraction be multiplied by an integer? How will you divide $\frac{2}{7}$ by 3, and why choose the method proposed? If the terms of a fraction be each increased by say 7, what would be the effect if it were a proper fraction, and what if it were an improper fraction? For what purpose are fractions reduced to a L. C. D.? How are fractions "multiplied together"? How "added together"? Add $\frac{2}{3}$ to $\frac{3}{4}$, and prove every step of the process. Is a multiplicand increased by multiplying it by $\frac{1}{7}$? What is multiplication? Can a number be taken a part of a time? If such a part of a number be taken as is shown by the fractional multiplier, how many times must such part be taken? (*Ans.* Once.) What is it to multiply by $\frac{3}{11}$? Multiply 4004 by $\frac{3}{11}$. Prove that $\frac{3}{4} \times \frac{5}{7} = \frac{15}{28}$. Define division so that the statement will apply to fractions. In how many ways may a fraction be divided by an integer? Which is generally the best way? How may the rule for dividing a fraction by a fraction be explained? Demonstrate the steps of the process of dividing $\frac{4}{7}$ by $\frac{3}{5}$. State the general principles of fractions. What mode of reasoning, not having direct reference to integral division, may be adopted in explaining the multiplication or division of a fraction? (See foot-note.) Before commencing the actual solution of a complex problem what should be done?

Section XIII. General Review of Integers and Fractions.

214. The Statement of Problems.

Before proceeding to solve a complex problem, a pupil should set forth each and every process by fitting signs. For example:

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Example 1.—If it be required to add $\frac{30}{5}$ to $\frac{42}{7}$ of 3 times 14, and then divide the result by $\frac{40}{5}$, it would be well to first represent the work thus: $\frac{(\frac{42}{7}\times 3\times 14)+\frac{30}{5}}{\frac{40}{5}}=?$ *Ans.* $32\frac{1}{4}$.

Example 2.—If the problem were as follows: What is that number of which 20 is 4 less than $1\frac{4}{7}$ times $\frac{120}{300}$ of the number itself? Then the conditions might be thus shown: $20+4=1\frac{4}{7}\times\frac{120}{300}$ of some number; and be solved thus:

$24=\frac{44}{7}$ of what?

$\frac{24}{44}=\frac{1}{7}$ of what? *Ans.* $\frac{168}{44}$, or $3\frac{36}{44}=3\frac{9}{11}$.

Example 3.—If the conditions of a problem were these, viz.: "To $\frac{3}{4}$ add $\frac{1}{4}$, divide the sum by a unit, to the quotient add $\frac{5}{7}$, and multiply the sum by 2, and state the result," the operation might be indicated as follows:

$\frac{(\frac{3}{4}+\frac{1}{4}+\frac{5}{7})\times 2}{1}=3\frac{3}{7}$. As $(\frac{1}{1}+\frac{5}{7})=\frac{12}{7}$, and $\frac{12}{7}\times 2=\frac{24}{7}$, or $3\frac{3}{7}$.

RULE.—*In formulating problems, use parentheses for groups of factors or for compound fractions. Simplify every expression, especially if it is to be increased or diminished by another. Write each simple quantity, whether product, quotient, sum, or difference, under its equivalent expression. Proceed with this second rank from left to right, and collect values—e. g., Example* 1, *supra. Write beneath:*

$$(6\times 3\times 14+6)\div 8=258\div 8=32\tfrac{1}{4}.$$

Example 4.—Simplify $\frac{3\frac{5}{9}-1\frac{7}{3}\text{ of }\frac{24}{25}}{11\frac{2}{3}\text{ of }\frac{9}{14}\text{ of }\frac{2}{5}}-\frac{4\frac{1}{3}-7\frac{5}{9}+3\frac{2}{3}}{\frac{5}{9}\text{ of }12}$.

1st num. $=3\frac{5}{9}-(\frac{15}{8}\times\frac{24}{25})=3\frac{5}{9}-\frac{9}{5}=$

$$\frac{160}{45}-\frac{81}{45}=\frac{79}{45}=1\frac{34}{45}.$$

1st den. $=\frac{35}{3}\times\frac{9}{14}\times\frac{2}{5}=\frac{5}{1}\times\frac{3}{2}\times\frac{2}{5}=3$.

Hence the 1st frac. $=\frac{79}{45}\div 3=\frac{79}{135}$.

The sec. num. $=4\frac{3}{9}+3\frac{2}{3}-7\frac{5}{9}=0$.

The sec. den. $=\frac{60}{9}=\frac{20}{3}$, or $6\frac{2}{3}$. Now $\frac{0}{6\frac{2}{3}}=0$.

$\therefore$ the given quantity $=\frac{79}{135}$.

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215. To find what Fraction one Number is of Another.

Note.—Divide that which is taken as a part by that of which it is a part. The divisor is, of course, the measure or standard, and whatever form the question may take, the divisor is preceded by the word *of*.

Exercise LIII.

1. How much must be added to $\frac{2}{17}$ of \$50 to make $\frac{1}{12}$ of $\frac{2}{5}$ of \$1,250?

2. How much has a person left, out of £20, who purchases \$19, each worth $\frac{108}{100}$ of $\frac{9}{40}$ of £1; 32 thalers, each worth $\frac{282}{200}$; and 18 Danish rix-dollars, each worth $\frac{13}{120}$ of a £?

3. A partner in a concern worth \$154,000 had a share of $\frac{2}{7}$. He sold half his interest, and spent the proceeds. At his death, he left to his 3 daughters equal portions of his property. What did each receive?

4. What part of 12 inches is 6 inches? 4 inches?

5. What part of $18\frac{1}{2}$ feet is $3\frac{1}{2}$ feet? $2\frac{1}{3}$ feet? $4\frac{1}{2}$ feet?

6. What part of $\frac{1}{2}$ of a foot is 3 inches?

7. What part of $\frac{1}{3}$ of a foot is $2\frac{1}{4}$ inches?

Remark.—Reduce the larger concrete number so that it will be of the same name as the smaller—*e. g.*, $\frac{1}{3}$ of a foot=$\frac{1}{3}$ of 12 inches, or 4 inches. Then $\frac{2\frac{1}{4}}{4}=\frac{\frac{9}{4}}{\frac{4}{1}}=\frac{9}{16}$. *Ans.*

8. What part of $\frac{4}{5}$ of a gallon is $\frac{7}{8}$ of a pint?

9. If a snail move $\frac{4}{5}$ of $\frac{1}{2}$ of $\frac{15}{16}$ of an inch in an hour, how long will it take him to travel $7\frac{3}{5}$ yards?

10. How many times can a pail holding $\frac{9}{10}$ of a quart be filled from $\frac{1}{3}$ of a barrel of $31\frac{1}{2}$ gallons?

11. From a piece of cloth measuring $35\frac{1}{2}$ yds. there were cut 2 coats, each requiring $4\frac{1}{5}$ yds.; 3 pairs of pantaloons, $2\frac{3}{4}$ yds. in each pair; and 4 vests $\frac{3}{4}$ of a yard in each; what remained?

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12. If the average weekly expenses of a man earning \$18 per week are \$3$\frac{1}{3}$ for rent, \$1$\frac{1}{2}$ for fuel, \$1$\frac{1}{4}$ for lights, \$7$\frac{1}{8}$ for food, \$2$\frac{1}{3}$ for clothing, \$$\frac{3}{7}$ for papers and books, and \$2$\frac{1}{8}$ for luxuries and medicines, the remainder being saved; what will he have at the end of 2$\frac{1}{2}$ years?

13. What will be left of 18 dollars, if from that sum there be taken the cost of 14 lbs. of sugar @ 13$\frac{1}{2}$ cts., $\frac{1}{2}$ lb. tea @ \$1.10, 2$\frac{1}{2}$ lbs. coffee @ 25 cts., 12 yds. muslin @ 18$\frac{3}{4}$ cts., and of 2 pairs shoes @ \$3$\frac{1}{4}$?

14. A merchant expended \$300 for rice and lard. He purchased 12 lbs. of the former at 7$\frac{3}{8}$ cts.; how many lbs. of lard at 12$\frac{1}{4}$ cts. did he purchase?

15. If 218$\frac{1}{2}$ lbs. of tallow are worth \$26.00, what is 4$\frac{1}{2}$ lbs. worth?

REMARK.—State the work thus:

The value of 1 lb. $= \frac{26}{218\frac{1}{2}}$; value of $4\frac{1}{2}$ lbs. $= \frac{9}{2} \times \frac{26}{4\frac{3}{2}}$.

16. Eleven men in a cricket match made a certain number of runs. A made $\frac{1}{8}$ of the whole number; B and C $\frac{1}{10}$ each; D, E, and F, $\frac{1}{20}$ each; and the rest added 126 runs to the score; G, H, I, and J each scored 5 times as many as K; what was the whole number of runs, and how many did each man make?

17. A man owned $\frac{1}{3}$ of a mill and then bought $\frac{1}{3}$ of the remainder; after this he sold $\frac{7}{16}$ of his interest; what share did he still possess?

18. A horse drank 4$\frac{1}{2}$ gallons of water from a tub, an ox 5$\frac{1}{6}$ gallons, a cow 2$\frac{1}{3}$ gallons, a sheep $\frac{1}{2}$ a gallon, and a dog lapped a half pint; if there were 16$\frac{1}{4}$ gallons at first, how much remained?

19. What cost $\frac{3}{4}$ of a pound of tea at \$$\frac{3}{4}$ a pound?

20. What will $\frac{5}{16}$ of a pound of coffee cost at \$$\frac{5}{16}$ a pound?

21. Divide the product of 4$\frac{3}{8}$ and 5$\frac{1}{3}$ by their sum; by their difference; and by their quotient.

22. A boy has a number of marbles of which he loses $\frac{3}{8}$ at

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play, gives away $\frac{5}{16}$ to one classmate, and $\frac{1}{4}$ to another; he then has 7 left; how many had he at first?

23. A grocer buys $4\frac{3}{4}$ cwt. of sugar, and retails it using a weight of $15\frac{1}{8}$ oz. for a pound; of how much does he defraud his patrons?

24. In dividing a drove of sheep, A took $\frac{3}{5}$ of them, and B the remainder, which was 1846 sheep: how many in the drove and how many did A take?

25. In a Fahrenheit thermometer there are 180° between freezing and boiling water; in a Réaumur, only 80°; and in the Centigrade, 100°; if a change of temperature of 20° Fahrenheit takes place, how many degrees would be shown on the Centigrade and how many on the Réaumur?

26. If a Réaumur should indicate a change of 20°, how many degrees of difference would be shown on a Fahrenheit; how many on the C. thermometer?

27. How many on the F. and R. would equal 20° on the C. thermometer?

28. If I buy 100 yds. of ribbon at 3 yds. for a dime, and another 100 yds. at 2 yds. for a dime, and sell all at 5 yds. for 2 dimes; how much do I lose?

Section XIV. Decimal Fractions.

216. A decimal fraction is one whose denominator is *understood* to be 1 with as many ciphers annexed as there are figures (following the decimal point) in its numerator. The *numerator only* is expressed, and one or more of its figures is preceded by a decimal point.

217. The figures following a decimal point are usually called *a decimal.*

In order to a thorough comprehension of operations in decimals, it may be well to reconsider these matters:

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218. I. That all numbers that exceed 9 are multiples of 10. Every result, whether sum, difference, product, or quotient must be so considered.

219. II. That in any line of figures the value of each digit depends chiefly on its *place*.

220. III. That in a line of similar figures, each expresses 10 times as much as the one on its right.

221. IV. That if the place of units be fixed, a units' figure should express 10 times as much as a similar figure standing on its right; it should express a hundred times as much as the second figure on its right; a thousand times as much as the third right-hand figure; and ten thousand times as much as the fourth right-hand figure.

222. NOTE A.—A power is the product of *equal* factors. Thus $4\times4\times4=$ the third power of 4 or 64. 10×10 or 100 is the second power of 10. $10\times10\times10\times10$ is the 4th power of 10. The first power of 10 is 10 itself, the first power of 6 is 6. The definition given does not *seem* to apply to first powers.

223. NOTE B.—The period which sets off the fraction from the integer is called the *decimal point*. By placing a period at the right of a units' figure and then writing similar figures at its right and left, thus, $\overset{3}{6}\overset{2}{6}\overset{1}{6}6.\overset{1}{6}\overset{2}{6}\overset{3}{6}$, we have a mixed number, *i.e.*, $6666\frac{666}{1000}$. The *names* of the opposite orders are somewhat *alike*, and the *values* of the figures are *reciprocal*. (*See Sec.* 210.)

The first figure on the left of the units is a multiple of the first power of 10; the second figure on the left, of the second power of 10, etc. The first figure on the right of the units is *divided* by the first power of 10; the second figure on the left is divided by the second power of 10; the third figure is divided by the third power of 10, etc. $6666.666=6000+600+60+6+\frac{6}{10}+\frac{6}{100}+\frac{6}{1000}$, and may be read, "six thousand six hundred and sixty-six, decimal, six hundred and sixty-six";

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or, "six thousand six hundred and sixty six units, and six hundred and sixty-six thousandths." That the decimal fraction may be considered as thousandths, will appear from the following:

$$\frac{6}{10} \times \frac{100}{100} = \frac{600}{1000}, \text{ and } \frac{6}{100} = \frac{6 \times 10}{100 \times 10}, \text{ or } \frac{60}{1000};$$

$$\therefore .666 = \tfrac{6}{10} + \tfrac{6}{100} + \tfrac{6}{1000} = \tfrac{666}{1000}.$$

224. The numerator only of a decimal fraction is written. It must be preceded by a decimal point. For each order there is a denominator understood, but, in practice, the denominator of the right-hand order is made the common denominator of each decimal figure. The first order of decimal figures is that on the *left*, and is called tenths; the second order is on the right of tenths, and is called hundredths; etc.

Point.	Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred thousandths.	Millionths.	Ten millionths.
.	2	2	2	2	2	2	2
	1st	2d	3d	4th	5th	6th	7th

225. In examining a decimal observe:

I. Its value as if there were no point.

II. The name of the right-hand order

226. To read a Decimal Fraction.

Rule.—*Read the numerator as an integer, and name its right-hand order.*

227. In reading a mixed number, recite the integer as *units*, or introduce the fraction with the phrase *and decimal*. If the integer end in *s*, no confusion need result. Thus 600.06 may be read, six hundred units and six-hundredths; or, six hundred, and decimal six-hundredths. (See Exer. LV.)

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Exercise LIV.

Read the following and then write them out as common fractions:

.5	.05	.12	.122	.012	.0122	.035	.3	.0019
.55	.99	.064	2.5	2.05	4.01	.417	1.9	.190
.01	1.1	1.05	.3001	.205	.3333	.27	.19	.1900
.2	.024	.0048	.506	.375	.005	.0608	.019	.19000

228. To write a Decimal Fraction.

Rule.— *Write the numerator so that the right-hand figure shall have the name of the denominator.* (See 229.)

For example, to write 42 ten-thousandths, the 2 must stand in the place of ten-thousandths, viz., in the fourth place, thus .0042.

229. One should consider, first, how many places are required; secondly, how many places the given numerator requires; and thirdly, the difference between these numbers of places, as indicating how many ciphers must be prefixed.

In the last example, the denominator indicates that four places in all are needed. The numerator plainly can occupy but two of these places. Hence two ciphers must be prefixed. A cipher must always be written in an order not called in reading.

Exercise LV.

Write out the following decimal numbers:

1–20. Three tenths. Twenty-five hundredths. Two hundred and forty-two thousandths. Nine ten-thousandths. Nine hundred-thousandths. Ninety-one ten-thousandths. One hundred and six millionths. Six hundredths. Six hundred-thousandths. Sixty-six ten-millionths. Fifty-four billionths. Four hundred-millionths. One hundred and six tenths. Two thousand and four hundredths. Seven units and five thousandths. Six hundreds and six hundredths. Eighty-two units

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and eighty-two millionths. Ninety units and ninety-one ten-billionths. Two hundred and one thousands and four-tenths. Six thousand units and forty-one millionths.

21. $\frac{7}{10}$; $\frac{51}{10000}$; $\frac{7}{10000000}$; $\frac{41}{1000}$; $\frac{2}{100}$.

26. $\frac{43}{100000}$; $\frac{29}{10000000}$; $\frac{410}{10}$; $\frac{2064}{100}$; $\frac{31}{1000}$.

31. $44\frac{3}{100}$; $2\frac{1}{10}$; $25\frac{3}{1000}$; $4\frac{1}{100}$; $\frac{3145}{100}$.

36. $\frac{247}{100}$; $\frac{351}{1000}$; $2\frac{10}{10000}$; $\frac{5}{100000}$; $\frac{214}{10}$.

41. What mixed common fraction will express 416 hundredths?

42. 91664 thousandths? 319 hundredths? 16 tenths? 51681 ten-thousandths?

46. Write 716 millionths; 209 ten-millionths; 304 ten-thousandths; 5 tenths; 5 thousandths.

230. In decimals, as in integers, the value expressed by a figure depends chiefly on its place. If by the use of ciphers, or by changing the location of the decimal point, a figure be made to occupy a place farther toward the right, its value is divided by 10, or 10 times 10, or by that power of 10 indicated by the number of places which it has been removed.

231. If a decimal figure be removed toward the left one place it will express ten times as much as before; if three places, then one thousand times as much as before, etc.

Exercise LVI.

Multiply the following decimals by ten:

1. 100.4; 5.168; 32.91; .0047; .8235.
6. 004.61; .243; .31; 4.586; 982.4.

Multiply the following by 100:

11. 49.68; .00418; 3.25; .519; 8.24.
16. .4003; .0496; .824; 0.04; .789.

Multiply the following by ten thousand:

21. .00496; .00042; .0031; .0426; .8146.
26. 3.9458; .056315; .4239; .0001; 1.00042

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Divide the following by one thousand:

31.	4.	90.	560.	204.	3569.
36.	11.4	97.3	256.9	845.7	32.5
41.	00.04	35.16	82.45	50.42	00.56
46.	0.351	0.049	0.005	8.216	5.973

EXERCISE LVII.

Read the following as common fractions:

1–12. .81; .081; .0096; .96; .00096; .000004; .0049; .25; 8.42; 36.9; .041; .00004182.

General Principles of Decimals.

232. I. Decimals arise from dividing ONE into ten equal parts, and from dividing each one of these *tenths* into ten equal parts, etc.

233. II. Decimals are written at the right of a separatrix or point, the numerator only being expressed.

234. III. The orders of decimals are numbered from the point, and the names correspond with those of whole numbers, except that one figure less is required for the expression of each name.

235. IV. Every cipher inserted between the separatrix and the digits of a decimal diminishes their value tenfold.

236. V. Ciphers annexed to a decimal do not alter its value, as they do not alter the position of the significant figures.

237. NOTE.—If the denominator be written, the expression is called a common fraction.

238. Any whole number may be made to take the form of a mixed number, and any decimal may be made to assume a larger denominator by the suffixing of ciphers. The effect is the same as when both terms of a fraction are multiplied by 10. (Fifth principle.)

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EXERCISE LVIII.

Reduce the following as directed:

1–20. .5 to thousandths. .025 to millionths. .004 to ten-thousandths. .0105 to billionths. .4 to hundredths. .003 to hundred-thousandths. .111 to ten-millionths. .12 to tenths. .104 to thousandths. .2 to millionths. 4.15 to ten-thousandths. .005 to hundred-millionths. .45 to billionths. .469 to ten-billionths. .00456 to millionths. .004 to hundred-thousandths. .3 to hundredths. .42 to ten-thousandths. .164 to tenths. .35683 to thousandths.

239. To reduce a Decimal to a Common Fraction.

Rule.—*Cancel the decimal point, supply the proper denominator, and reduce the expression to its lowest terms.*

EXERCISE LIX.

Reduce the following to common fractions:

.125; .034; .56; .25; .0025; 1.04; 2.5; 1.16; .08; .625; 3.5; .48; .0005; 1.225; .36.

Note.—For other examples, see the preceding articles.

240. To reduce a Common Fraction to an equivalent Decimal.

Example 1. $\frac{4}{5}$ equals what decimal? *Process:* 5)2.0
Explanation. 2=2.0, or 20 tenths. .4

$\frac{1}{5}$ of 20 tenths is 4 tenths.

Rule.—*Annex ciphers to the numerator, and divide by the denominator, and point off in the result as many figures as there are ciphers annexed.*

Note.—The annexation of ciphers to the numerator multiplies the fraction, and would, of course, make too large a quotient if the result were not corrected by the subsequent pointing off. Thus the 2 in $\frac{4}{5}$ is made ten times 2; but the quotient 4 is subsequently converted into one tenth of 4, by way of correction.

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Exercise LX.

Reduce the following to decimals:

1–40. $\frac{3}{5}$ of $\frac{9}{30}$; $\frac{5}{75}$; $\frac{2}{16}$; $\frac{6}{16}$; $\frac{3}{4}$; $\frac{1}{4}$ of $\frac{1}{12}$; $\frac{2}{5}$ of $\frac{5}{4}$; $\frac{9}{18}$; $\frac{2}{4}$; $\frac{2}{10}$; $\frac{5}{16}$; $\frac{1}{32}$; $\frac{5}{64}$; $\frac{3}{8}$; $\frac{9}{128}$; $\frac{12}{8}$; $\frac{10}{16}$; $\frac{18}{32}$; $\frac{4}{25}$; $\frac{7}{8}$; $\frac{1}{250}$; $\frac{16}{128}$; $\frac{14}{20}$; $\frac{9}{80}$; $\frac{17}{16}$; $\frac{3}{40}$; $\frac{13}{320}$; $\frac{9}{160}$; $\frac{124}{240}$; $\frac{41}{120}$; $\frac{1.12}{8}$; $\frac{2.10}{16}$; $\frac{3.4}{32}$; $\frac{16}{64}$; $\frac{1.8}{128}$; $\frac{26}{40}$; $\frac{21}{80}$; $\frac{84}{128}$; $\frac{35}{64}$; $\frac{91}{160}$.

Repetends.

241. When a numerator with ciphers annexed will not exactly contain the denominator, it will be found that the same figure or sets of figures will recur constantly. Such results form *circulating* or *infinite* decimals, and the repeating figure or figures are called **Repetends.**

242. When the denominator of a common fraction in its lowest terms contains any factor other than *two* and *five* the fraction has no exact equivalent *decimal,* because its denominator is not related to 10.

243. It is generally sufficient to carry on the ordinary division of the numerator from three to six decimal places, as the result then obtained will differ from the true result by less than a thousandth or a millionth. If strict accuracy is desirable, use common fractions.

(For practical discussion of Repetends see the Second Part.)

Section XV. Addition of Decimals.

244. Let it be required to add 2.146, 3.1, 9.46845, and .004. In adding common fractions, the quantities are reduced to a common denominator, and the sum of the numerators is then written over the common denominator. Let each of the above

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take five decimal places; then they each will have 100000 for denominator, thus:

$$2.14600 = 2\tfrac{14600}{100000},$$
$$3.10000 = 3\tfrac{10000}{100000},$$
$$9.46845 = 9\tfrac{46845}{100000},$$
$$.00400 = \tfrac{400}{100000},$$

$$14.71845 = 14\tfrac{71845}{100000};$$

and their sum may be pointed off as hundred-thousandths.

In decimals the scale is the same as in integers. The introduction of ciphers, as above, is unnecessary, the value of each figure being known by its position. Therefore the decimals last added might have been written thus:

2.146
3.1
9.46845
.004
14.71845

To add Decimals.

Rule.—*Write the numbers so that the tenths will form a column. Add similar orders as in Addition of Integers. Point off as many places in the amount as equal the largest number of places found in any of the addends.*

245. Dimes, cents, and mills, in U. S. currency, are expressed decimally. Dollars are integers, dimes are tenths, cents are hundredths, and mills are thousandths. Thus 12 dollars 14 cents and 7 mills are written $12.147.

TABLE OF U. S. CURRENCY.

10 mills (m.)	make	1 cent,	marked	ct.
10 cents	"	1 dime,	"	d.
10 dimes	"	1 dollar,	"	$
10 dollars	"	1 eagle,	"	e.

One dollar is regarded as the unit.

246. Note.—In reading sums of U. S. currency, dollars, cents, and mills are the only denominations in common use. The figures on the left of the decimal point

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are read together as a whole number, and called dollars. The two figures nearest the decimal point on the right are read together, and called cents. One dime being ten cents, it is evident that the dimes may be correctly read as ten, twenty, thirty cents, etc. $417.64 is read four hundred seventeen dollars and sixty-four cents; $164.718 is read one hundred sixty-four dollars, seventy-one cents, eight mills; $4.3257 is read four dollars, thirty-two cents, five mills, and seven-*tenths* of a mill. If there are two figures at the right of mills, they may be called *hundredths* of a mill, etc. In common business, if there is a fraction of a cent equal to five mills or more, it is customary to consider it as 1 cent. Cents are often read as hundredths of a dollar. If the number of cents be less than 10, a cipher must be prefixed. A decimal point must separate dollars and cents.

Exercise LXI.

1. $38.3+371.27+5.40+68.29+4.15=$?

2. $2\frac{1}{2}+3\frac{1}{5}+4\frac{1}{8}+5\frac{1}{16}+3\frac{3}{4}=$?

3. Bought a hat for $5, a vest for $3.50, a coat for $15.50, and a pair of boots for $4.25. What is the cost of the whole?

4. Admit that I am in debt to one man $14.27; to another, $4.325; to another, $37.04; to another, $37; and to another, 25 cents. How much do I owe in all? *Ans.* $92.885.

5. If I walk on Monday 24.2 miles; on Tuesday, 31.25 miles; on Wednesday, 19.5 miles; and on Thursday, 26 miles; how many miles do I walk in the four days?

Ans. 100.95.

6. If a cow is bought for $31.45, a horse for $86.375, a wagon for $62.0625, and a set of harness for $19.30, what is the cost of the whole? *Ans.* $199.1875.

7. Add 324.48, 470071.3, 8641.3062, and 2571.0003.

8. $14.32+108.96+4341.007+98400.13+45.=$?

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9. 2.7+30.84+75.1+126.414+3.7+.028+10.09=?
10. .362+.47+9.322+2.908+161.56+1142.25=?
11. 7.15+143.002+8.075+999.999+24+496.=?
12. What is the sum of \$46.29, \$128.10, \$4.75, \$659.04, \$3.141, and \$260.48?

13. Bought a spelling-book for 25 cents, a dictionary for \$1.125, a reader for 75 cents, a slate for 31 cents, a pencil for 1 cent, and a penknife for 50 cents. What do they all amount to? *Ans.* \$2.945.

14. Add 9 cents, 12 cents, 24 cents, 56 cents, 87 cents, 72 cents, 41 cents, 22 cents, and 3 cents. *Ans.* \$3.26.

15. Suppose I am indebted to A twenty-seven dollars thirty cents, to B sixty dollars fifty cents and nine mills, to C fifty dollars ninety-eight cents, to D one hundred dollars seven mills, to E seven dollars four cents and eight mills, to F thirty dollars seventy-five cents, to G forty-eight dollars twenty cents, and to H seventy-three dollars twenty-seven cents; how much is the amount of my debts?

Ans. \$398.064.

16. $2\frac{1}{2}+3\frac{1}{5}+4\frac{1}{8}+5\frac{1}{16}+1=$?
17. $\frac{307}{1000}+2\frac{1}{24}+17\frac{1}{8}+\frac{26}{1000}+.16=$?
18. $2\frac{5}{8}+.125+4\frac{8}{40}+13\frac{1}{150}+2.3=$?
19. 2.067+27.09+10.15+35.560+256.1=?
20. $\frac{6}{10}+\frac{14}{100}+\frac{250}{1000}+\frac{31}{10000}+\frac{1682}{100000}=$?

Section XVI. Subtraction of Decimals.

247. To subtract one Decimal Number from another.

Rule.—*Write the number having the smaller integral part under the other, so that tenths may stand under tenths, hundredths under hundredths, etc. Subtract as in whole numbers, and point off as in addition of decimals.*

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Illustration.—From 91.73 take 12.138

Operation.—91.73
12.138

79.592

Blank orders may be considered as if occupied by ciphers.

Exercise LXII.

Express the differences of the following:

1–16. 714.5—13.916; .145—.09684; 2.—.2; 13.—.013; 1.6—.016; .18—.0018; $12\frac{6}{100}-3\frac{7}{10}$; \$17.25—\$16.045; \$250.—\$137.14; \$116.—\$112.455; .84—.002; .004—0001; 11.11—.1111; 2.34—.0234; .025—.018; $78\frac{9}{10}-5\frac{16}{1000}$.

17. \$10—(\$4.62+.75+.125+.25+.03).

18. 25—(4.50+11.50+.75+2.10+.04).

19. $\$47.87\frac{1}{2}$—(\$10.75+$\$8.62\frac{1}{2}$+\$12.25).

20. On a note for \$100 the following payments were made, viz., \$12.20, \$15, and \$37.50. What is due?

Section XVII. Multiplication of Decimals.

Multiply .25 by .5. As $\frac{25}{100}\times\frac{5}{10}=\frac{125}{1000}$, $.25\times.5=.125$.

248. It is evident that the denominator of the **product of two decimals** must contain all the factors of the denominators of the decimals. Hence the number of decimal figures in the product must equal the sum of the numbers of places in both factors. As .25 contains two places, and .5 one place, their product must contain three places; for the number of decimal figures is always dependent on the denominator.

When either or both Factors are Decimals.

Rule.—*Multiply as in whole numbers, and point off in*

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the product as many decimal figures as there are in both factors, prefixing ciphers when necessary.

Example 1.—5.236 × .008 Regarded as integers, the product would be 41888, a number having but *five* figures. Now, thousandths into thousandths produce millionths, which, written decimally, require 6 decimal figures; ∴ 41888 millionths must be written thus: .041888.

NOTE.—In solving the following examples the learner will recall what has been said respecting the reduction of common fractions to decimals, and also the explanations concerning the decimal form of U. S. currency.

EXERCISE LXIII.

1. What will 5.5 yards of linen cost, at 62½ cents a yard? 5.5 × .625?
2. Req. cost of 8½ lbs. of sugar, at 12½ cents a lb.
3. Req. cost of 15.4 acres, at $37.42 an acre.
4. Multiply 2461 by .0529 *Ans.* 130.1869.
5. Multiply 7.353 by 3.5. *Ans.* 25.7355.
6. Multiply 4.001 by .004. *Ans.* .016004.
7. Multiply .00071 by .121. *Ans.* .00008591.
8. Multiply 4.1 by 1.42. *Ans.* 5.822.
9. What is the value of 253.5 bushels of wheat at $1.25 a bushel? *Ans.* $316.875, or $316.87½.
10. How much will 5 bushels of potatoes cost, at 50 cents a bushel? *Ans.* $2.50.
11. What will 8 cords of wood cost, at $4.50 a cord? *Ans.* $36.
12. What cost 94 bushels of oats, at 63 cents a bushel?
13. Bought 37 oxen for $52 each; what did they cost? *Ans.* 1924.
14. What do 60 firkins of lard cost, at $7.14 a firkin?

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15. What cost a farm containing 125 acres, at $36.50 an acre? *Ans.* $4562.50.

16–18. 27.4×3.56; $373.3 \times .0675$; $1.032 \times .00421 = ?$

19, 20. $.042 \times .8$; $2.673 \times 1.02 = ?$

21. What is the cost of 2481 pounds of sugar, at 3 cents a pound?

In the last example, the 3, which we use as the multiplier, must be regarded as a decimal figure standing in the place of hundredths; therefore the product must have two figures for decimals or cents. $(.03 \times 2481 = 2481 \times .03.)$

22. What is the value of 25,415 pounds of cotton, at 17 cents a pound?

23. How much must be paid for 648 pounds of raisins, at 18 cents a pound?

24. At 4 cents a mile, what must I pay for riding 362 miles?

25. What must I pay for 36.25 pounds of beef, at 16 cents a pound?

26. At $5\frac{3}{4}$ a bbl., what cost $87\frac{1}{2}$ bbls.

27. What would buy 300 quills, if the cost of each should be 4 mills?

28. Req. cost of $48\frac{1}{2}$ lbs. cheese, at 17 cents a lb

29. Req. cost of $29\frac{1}{4}$ lbs. of sugar, at $14\frac{3}{4}$ cents.

30. Req. cost of 60 lbs. tea, at $1.12\frac{1}{2}$.

31. Req. value of 50 bush. wheat, @ $1.87

Req. values of the following:

32. 45 lbs. of flour, at $3\frac{1}{2}$ cents per lb

33. $13\frac{1}{2}$ yds. of silk, at $2.25.

34. $35\frac{3}{4}$ lbs. nuts, at $28\frac{1}{2}$ cents.

35. $12\frac{1}{2}$ lbs. crackers, @ $10\frac{1}{2}$ cents.

36. $60\frac{3}{4}$ yds. cloth, @ $5\frac{1}{2}$.

37. $26\frac{1}{2}$ dozen eggs, at $1\frac{1}{2}$ cents for each egg.

38. $15\frac{3}{4}$ dozen eggs, at $2\frac{1}{4}$ cents for each egg.

39. $2\frac{1}{2}$ lbs. nutmegs, at 25 cents an ounce.

40. $1\frac{3}{4}$ lbs. indigo, at 18 cents an ounce.

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Method of Contracting Multiplication of Decimals.

249. As decimal figures beyond the fourth place are generally of little practical consequence, it is desirable to know how to **contract operations in multiplication of decimals.**

Ill.—.785497×2.54, retaining 4 places in the product.

.7854'97	It will be observed that the units figure
2.54	of the multiplier was employed first, and
———	that the fifth figure of the multiplicand was
15709	considered no further than to note what
3927	must be "carried" to the product of the
314	fourth figure.
———	In multiplying by the 5 tenths, the *third*
1.9950	figure of the upper number was first multiplied; and in taking the product of the 4 hundredths into the multiplicand, its *second* figure was used. Now the product of a figure of the second order into one of the same order gives a number of the *fourth* order, beyond which it was not proposed to go.

a. 4.965318×2.643=13.1233 | 35474.

b. **Reverse** the figures of the multiplier. In finding each partial product commence with the figure *immediately above* the multiplying figure. Thus: 2×3; 6×5; 4×6; 3×9.

Req. prod. of 1.44619 by .423 to 4 places. *Ans.* .6116.

Req. prod. of .3785 by .4673 to 5 places. *Ans.* .17687.

b. Contraction.

4.9653'18
346.2
———
99306
29791
1986
148
———
13.1231

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Req. prod. of 4.356126 by 52.34 to 3 places. *Ans.* 227.998.

Contraction.

4.3561'26
43.25

217806
8712
1306
174

227.998

In this case, as the 5 of the multiplier stands for tens, the first figure of the multiplicand taken is ten thousandths figure. The units, 2, is multiplied into the third order above; the tenths of the multiplier, 3, is combined with the second order of the multiplier; and the hundredths, 4, is multiplied into the tenths above.

Req. the prod. of 92.4023553 by 67.14986 to 4 places.

Ans. 6204.8037.

Section XVIII. Division of Decimals.

250. As the dividend is always equal to the product of the divisor and the quotient, the number of decimal places in the quotient counted together with those in the divisor, must be equal to the number of decimal figures in the dividend.

251. BRIEF RULE FOR DIVISION OF DECIMALS.—*Divide as in whole numbers, and point off as many places in the quotient as, counted with those in the divisor, will make as many as there are in the dividend.*

Beginners will find it to their advantage to consider the following instances:

252. I. When there are more places in the dividend than there are in the divisor; as, $2.3905 \div .07 = 34.15$.

253. II. When the number of places in the dividend equals the number of places in the divisor $239.05 \div 07 = 3415$.

254. III. When there are more places in the divisor than in the dividend. $2390.5 \div .07 = 34150$. for, $2390.50 \div .07 = 34150$. ($2390.5 \div 7 = 341.5$; $2390.5 \div \frac{7}{100} = 34150$).

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The annexation of the cipher in the last example reduced both divisor and dividend to a common denominator. In explaining the division of common fractions, it was shown that the process was equivalent to the division of one numerator by the other, after the fractions were reduced to a common denominator. Thus $\frac{2}{3} \div \frac{5}{6} = \frac{4}{6} \div \frac{5}{6} = \frac{4}{5}$.

It must not be forgotten that *annexing ciphers to a decimal does not change its value.*

255. GENERAL RULE FOR DIVISION OF DECIMALS.

Make at least as many places in the dividend as there are in the divisor by annexing ciphers if necessary.

Divide as in whole numbers.

If there be a remainder after bringing down all the figures of the dividend, annex ciphers to it and continue the division as far as may be thought proper. The ciphers thus annexed represent additional decimal places in the dividend.

Finally, point off as many decimal places in the quotient as, counted with those in the divisor, will equal the number of decimal figures in the dividend.

EXERCISE LXIV.

1. 274.855÷.7853= ?
2. 11÷55=?
3. .9÷450=?
4. 1.855÷26.5=?
5. 2406.976÷572=?
6. 11.7348÷254=?
7. 4066.2÷648=?
8. 637.531÷4.72=?
9. 3.672÷81=?
10. 42.665÷33.5=?
11. 1696.3200÷4560=?
12. .000140215÷.0005=?
13. 25.7690÷3.65=?
14. 3.33266÷593=?
15. .035624÷.244=?
16. 148.63÷4.21=?
17. 2142÷3.2=?
18. 2.00385÷931=?
19. 1.97064÷322=?
20. 2093.9460÷38.63=?

NOTE.—For brief solutions see **271.**

21. If 918 persons contribute $36.72, what does each average?

22. What cost 1 lemon, if 275 cost $2.475. $.009.

23. At $.75 per bush., how many bushels of oats can be bought for 141 dollars? 188.

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24. $67.50 is expended for butter at 37½ cents a pound; how many pounds are bought? 180.

25. If a clerk receive $650 a year, how much does he receive a day, excluding Sundays? $2.076.

26. If 11.5 cords of wood be bought with $24.475; how much does a cord cost? $2.12+

27. What is the mileage when $6.75 is paid for riding 225 miles? 3 cents.

Find the quotients of the following expressions:

28–38. $\frac{34.65}{8.75}$; $\frac{126.5621}{18.75}$; $\frac{18.5625}{148.5}$; $\frac{28.985}{1.87}$; $\frac{270.29}{1.79}$; $\frac{6.9564}{856}$; $\frac{11.25}{.45}$; $\frac{11.25}{4.5}$; $\frac{11.25}{45.}$; $\frac{112.5}{.45}$; $\frac{1.125}{4.5}$.

39–47. How many dollars in each of the following sums? 42879 cents; 63800 cents; 451083 mills; 764 cents; 104693 mills; 113456 cents; 3982 dimes; 4269 half-dimes. In 5 eagles, 5 half-eagles, $5, 5 half-dollars, 5 quarter-dollars, 5 dimes, 5 half-dimes, and 5 cents counted together, forty coins in all, how many dollars?

For additional questions refer to the "Table of Decimals."

NOTE.—Review carefully the matter of multiplying and dividing decimals by 10, 100, 1,000, etc. After presenting the subject of denominate numbers, additional practical questions will be furnished. The discussion of common fractions and decimals should occupy a very large share of the time devoted to arithmetic, and no pains should be spared to secure facility and accuracy in calculations made with fractional numbers. After solving the problems under Art. 271, the class should receive very many brief practical lessons, oral and written, on questions adapted to show the relations of decimal and common fractions, especially in their application to currency questions. The more difficult problems succeeding Division should be thoroughly analyzed.

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256. *Table of Decimals for Miscellaneous Problems.*

a	.06	*f*	.216	*k*	5.236	*p*	.528
	.02		.482	*l*	1.728		.484
	.735	*g*	.112		1.196	*q*	.3333
b	.801		.299		277.274		7.44
	.017	*h*	.608	*m*	.231	*r*	735.812365
c	56.345		2.006		.282	*s*	25.72046
	42.306		.032		2.688	*t*	61.0008719
	7.615	*i*	4.545	*n*	61.028	*u*	7.7888
d	114.568		7.854		5.760	*v*	4.4444
e	12.372	*j*	31.416	*o*	4.865	*w*	6.789023

Exercise LXV.

Examples illustrative of the use of the *table:*

1. Add *r*, *s*, *t*, *u*, *v*, and *w*.
2. Add *k* and the six following numbers.
3. Add *a* and the five following numbers.
4. Add all that precede *n*.
5. Add all that precede *i*.
6. Add *e* and the next four.
7. Add *p* and the next five.
8. Add the first column.
9. Add the third column.
10. Add the last column.

1. From *a* subtract the next number.
2. From *b* subtract the next.
3. From *c* subtract the next.
4. From *d*, the next.
5. From *j*, the next.
6. From *p*, the next.
7. From *r*, the next.
8. From *u*, the next.
9. From *w* take *v*.
10. From *s* take *u*.

Ten Problems in Multiplication.

1. Multiply *a* by *b*; 2. *c* by *d*; 3. *e* by *f*; 4. *g* by *h*; 5. *i* by *j*; 6. *k* by *l*; 7. *m* by *n*; 8. *o* by *p*; 9. *q* by *r*; 10. *s* by *t*.

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TEN PROBLEMS IN DIVISION.

1. Divide c by 3. 2. d by h; 3. e by 4.2; 4. q by a; 5. j by 6; 6. n by 2.2; 7. m by l; 8. p by 8; 9. j by 4; 10. l by 12.

The number of problems in Numeration, Multiplication, etc., which such a table affords is limitless.

257. *Questions to follow Decimal Fractions.*

Is the notion of a fraction always conveyed by the term decimal? What does the word "decimals" generally mean? What denominator is every decimal fraction understood to have? What purpose does 10 serve in Notation? What may the denominator of a Common Fraction be? What part of 4, standing in the 3d place on the left of the separatrix, is 4 standing in the 3d place on the right of it? What is the use of ciphers employed as prefixes in decimals? What as suffixes? How is a decimal fraction multiplied by 10? How divided by 10? In what respect, when they are to be added, are fractions of a decimal form more convenient than common fractions? How is a common fraction converted into a decimal? How a decimal into a common fraction? Reduce $\frac{1}{4}$ to a decimal and show why the result is correct? Where must the point be placed in Addition of Decimals? Why? Where should the point stand in a product, the factors being decimals? Take the product of .03 by .03, and prove that the point should be followed by a certain number of figures. In what way is the multiplication of the (understood) denominator of a decimal shown? What is the rule for Multiplication of decimals? For Division of Decimals? Why must U. S. money in dollars and cents be regarded as decimal? What is the unit of U. S. money? May all Common Fractions be accurately expressed in decimals? What are Repetends? When will a Common Fraction produce a Repetend? What should be written for the denominator of a Repetend in order to ex-

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press it accurately? (As many 9s as there are decimal figures.) Do we usually care to express Repetends exactly in the form of Common Fractions? Why not?

DENOMINATE NUMBERS.

258. Denominate Numbers are **concrete numbers,** expressing one or more units of time, value, weight, capacity, length, or surface.

259. A **simple denominate number** shows but one kind of units, as, 10 pounds, 3 yards.

260. A **compound denominate number** expresses **several denominations** of a **similar** nature, as 2 pecks, 1 quart, 1 pint.

261. Denomination means **name,** and is expressive of a *concrete number.*

262. NOTE—If we are counting sheep or oranges, our unit is one of the things before us; but if we are measuring length, as of a river, there are no natural divisions into parts by means of which we can count. We must fix on some length as a means of comparison. This, whether it be a foot, or a yard, or a metre, we call our **unit of length.** So if we would measure a cask or a vat, as there exists no separate parts or objects to be counted, we must prepare and use some unit as a means of comparison. We then have reduced the question *How much?* to the form *How many?* which is the only one to which number offers an answer. If concrete numbers were all of one denomination, as those of length, say in feet, those of capacity, in quarts, they would come under the common rules. Or if their different denominations were related by the scale of 10,

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as in U. S. currency, then the ordinary rules with those for decimals would serve; but as such is not usually the scale, it is necessary to memorize the tables which connect the different units together. Time, weight, and measure are referred to standards fixed by Government. A coin must be of a certain weight and purity; a gallon must be exactly equal to the standard gallon, no larger, no smaller.

263. The names of coins, weights, etc., are expressed by **abbreviations** called **signs.** These are, usually, initial letters of the English or Latin names. Thus £, *s.*, *d*, and *q.*, stand for *libra, solidus, denarius,* and *quadrans,* old Roman coins. *q.* has given place to ¼.

264. Each denomination **must stand by itself,** the lower at the right. If a person were to attempt to express quarts and gallons by the number 231, the reader would not know whether the writer meant to show 2 gallons and 31 quarts, or 23 gallons and 1 quart. U. S. money and the French measures are **exceptions** to this rule, as respects the separation of the numbers.

265. Compound numbers are **similar** in kind when they are derived from the *same unit.* Denominations of the same table are similar. So, too, those which may be converted into terms of another are similar; as gallons and bushels.

266. A **table** shows the comparative values of similar numbers.

267. A **standard unit** is that of which the numbers in the table are fractions or multiples. Thus, in U. S. money, the dollar is the standard unit; in cloth measure, it is the yard; in land measure it is the acre.

268. NOTE.—Foreign money of certain kinds has been legalized by Congress, among others, the gold coins of Brazil, Great Britain, Columbia, Mexico, Portugal, and Spain, and the silver coins of France, Mexico, Peru, and Chili; their values having been fixed in accordance with their purity and weight.

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269. Table of U. S. Money.

10 mills (marked m.)	make	1 cent.	ct. or c.
10 cents	"	1 dime.	d.
10 dimes	"	1 dollar.	$ or dol.
10 dollars	"	1 eagle.	e.

Eagle.	Dollars.	Dimes.	Cents.	Mills.
1 =	10 =	100 =	1000 =	10000
	1 =	10 =	100 =	1000

Note.—The use of U. S. money has been exemplified by the numerous examples already presented. Calculations may frequently be made more rapidly by

The use of Aliquot Parts.

270. Aliquot means **dividing exactly.** The aliquot parts of a dollar are said to be

50c. $=\frac{1}{2}$	of $1	$12\frac{1}{2}$c. $=\frac{1}{8}$	of $1
$33\frac{1}{3}$c. $=\frac{1}{3}$	" "	10c. $=\frac{1}{10}$	" "
25c. $=\frac{1}{4}$	" "	$6\frac{1}{4}$c. $=\frac{1}{16}$	" "
20c. $=\frac{1}{5}$	" "	5c. $=\frac{1}{20}$	" "
$16\frac{2}{3}$c. $=\frac{1}{6}$	" "	4c. $=\frac{1}{25}$	" "

These numbers give the simple integral quotients 2, 3, 4, 5, 6, 8, 10, 16, 20, and 25.

271. To find the Cost of a Number of Articles each of which is valued at an Aliquot Part of a Dollar.

Rule.—*Multiply by that fraction of a dollar which represents the given part.*

Exercise LXVI.

1. At 25 cts. a yd., what will $37\frac{3}{4}$ yards of muslin cost?

Illustration.

At $1 per yd. = $37.75

At 25 cts. or $\frac{1}{4}$ = $ 9.43$\frac{3}{4}$

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2. At 75 cts. a lb., what cost 62½ lbs. of butter? *Ans.* $46.875.

At $1 the cost	=	$62.50
(Subtract) ¼	=	15.625
At $¾	=	$46.875

3. At 87½ cents a lb., what is the value of 4 lbs. 10 oz. of tea? (Art. 271.) *Ans.* $4.05.

For 4 lbs. 10 oz. = 4 10/16, or 4⅝ lbs., and, at $1 = 4.625,
At $⅛ = .578,
At $⅞ = 4.047.

Find the cost of the following:

4. 376 bushels of wheat at $1.12½ a bush.; also at $2.20.
6. 150 lbs. of sugar at $8.62½ a hundred lbs.; also at $7.75.
8. 27½ gallons brandy at $4.25 a gallon; also at $2.50.
10. 45⅝ lbs. butter at 33⅓ cents a lb.; also at 50 cts.
12. 75 bushels of rye at $1.33⅓ a bushel; also at 66⅔ cts.
14. 12 quarts wine at 87½ cts. a quart; also at 37½ cts.
16. 29¾ yards calico at 20 cts. a yard; also at 25 cts.
18. 160 lbs. beef at 6¼ cts. a lb.; also at 12½ cts.
20. 21½ gallons sirup at 56¼ cts. a gallon; also at 25 cts.
22. 64 melons at 25 cents each; also at 20 cts.
24. 180 pencils at 16⅔ cents each; also at 36⅔ cts.
26. 27 dozen eggs at 33⅓ cts. a dozen; also at 25 cts.
28. 480 ounces of seed at 5 cents an ounce; also at 10 cts.
30. 492 oz. of nutmegs at 8⅓ cts. an ounce; also at 16⅔ cts.
32. 964 cows driven at 62½ cents a head; also at 87½ cts.
34. 98 lbs. wool at 36⅔ cts. a lb.; also at 20 cts.
36. 128 quarts berries at 8⅓ cents a quart; also at 12½ cts.
38. 288 dozen clothes-pins at 6¼ cts. a doz.; also at 12½ cts.
40. 908 lbs. coffee at 33⅓ cts. a lb.; also at 25 cts.

(See further applications of aliquot parts in succeeding tables.)

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TABLES OF CURRENCIES.

1. U. S. Currency. (See Article 269.)

2. English Currency.

272.

4 farthings (qr.)	make	1 penny,	d.
12 pence	"	1 shilling,	s.
20 shillings	"	1 pound,	£.
21 shillings	"	1 guinea.	
5 shillings	"	1 crown.	

£1 is of the *real* value of $4.8665 (Sec. Treas. 1875).

Aliquot parts: 6s. 8d. $= \frac{1}{3}$ of a £. 3s. 4d. $= \frac{1}{6}$ of a £.
5s. $= \frac{1}{4}$ " 2s. 6d. $= \frac{1}{8}$ "
4s. $= \frac{1}{5}$ " 1s. 8d. $= \frac{1}{12}$ "

£1 = 20s. = 240d. = 960qrs.

3. French Currency.

273.

10 centimes (cent.)	make	1 decime,	d.
10 decimes	"	1 franc,	fr.

5.18 fr. = $1. 1 fr. gold = 19.3 cents.
1 fr. silver = 18.2 cents.

TABLES OF WEIGHTS.

4. Troy Weight.

274.

24 grains (gr.)	make	1 pennyweight,	dwt.
20 pennyweight	"	1 ounce,	oz.
12 ounces	"	1 pound,	lb.

1 lb. = 12 oz. = 240 dwts. = 5760 grs.

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5. Avoirdupois Weight.

275.

16 drams	(d.)	make 1 ounce,		oz.
16 ounces		" 1 pound,		lb.
25 pounds		" 1 quarter,		qr.
4 quarters		" 1 hundredweight,		cwt.
20 hundredweights		" 1 ton,		t.

1 = 20 = 80 = 2000 = 32000 = 512000.

6. Apothecaries' Weight.

Used in *mixing* medicines. (Drugs in bulk by avoirdupois weight.)

276.

20 grains	(gr.)	make 1 scruple,	℈
3 scruples		" 1 dram,	ʒ
8 drams		" 1 ounce,	℥
12 ounces		" 1 pound,	lb.

1 lb. = 12 ℥ = 96 ʒ = 288 ℈ = 5760 gr.

277. 7. French Weights.

The unit is the GRAM. A cubic centimetre = 15.432 grains.

	No. Grams.	Volume of Water.	Avoirdupois lbs.
Millier, or *stere*,	1000000	1 cubic metre, .	2204.6 pounds.
Quintal, . . .	100000	1 hectolitre, . .	220.46 "
Myriagram, .	10000	10 litres, . . .	22.046 "
Kilogram, kilo,	1000	1 *litre* (1 quart),	2.2046 "
Hectogram, .	100	1 decilitre, . .	3.5274 oz.
Decagram, . .	10	10 cu. centimetres	.3527 oz.
GRAM, . . .	1	1 cu. centimetre,	15.432 grains.
Decigram, . .	$\frac{1}{10}$	$\frac{1}{10}$ do. . .	
Centigram, . .	$\frac{1}{100}$	$\frac{1}{100}$ do. . .	
Milligram, . .	$\frac{1}{1000}$	$\frac{1}{1000}$ do. . .	

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278. *Summary of the Metric System.*

Grades:	4	3	2	1	0	1	2	3
Values:	10000	1000	100	10	1	.1	.01	.001
Prefixes:	Myria.	kilo.	hecto.	deca.	**Unit.**	deci.	centi.	milli.

m.=39.37 in. . . . . **Metre.**
l.=61.03 cu. in. . . . . **Litre.**
kg.=2⅕ lbs. . . . . **Gram.**
a.=119.6 sq. yds. Hectare, **Are,** centiare.
fr.=19.3 cts. (intrinsic value) . **Franc.**

(For other particulars, see problems in Exercises 69, 73, 75, 78, 81, etc.)

MEASURES OF LENGTH.

8. *Linear Measure, English.*

279.

12 inches (in.)	make	1 foot,	ft.
3 feet	"	1 yard,	yd.
5½ yards	"	1 rod,	rd.
40 rods	"	1 furlong,	fur.
8 furlongs	"	1 mile,	mi.
69½ miles	"	1 degree,	deg.

1 mi.=8 fur.=320 rd.=1760 yd.=5280 ft.=63360 in.

280. *9. Linear Measure, French.*

10 millimetres (mm.)	make	1 centimetre, cm.,	=.3937 in.
10 centimetres	"	1 decimetre,	=3.937 in.
10 decimetres	"	1 METRE,	=39.37 in.
10 metres	"	1 decametre,	=393·7 in.
10 decametres	"	1 hectometre,	=328 ft. 1 in.
10 hectometres	"	1 kilometre,	=3280 ft.10in.
10 kilometres	"	1 myriametre,	=6.2137 mi.

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The metre is the *unit*. It = 39.37 inches.
Minute measurements are expressed in mm.
Common measurements in m., and distances in kil.

10. Linear Measure, Surveyor's.

281.

7.92	inches	make	1	link, l.
25	links	"	1	rod, rd.
4	rods	"	1	chain, ch.
80	chains	"	1	mile. m.

11. Measure of Surface or Area, English.

282.

625	square links	make	1	pole, p.
16	poles	"	1	square chain, sq. ch.
10	square chains	"	1	acre, a.
640	acres	"	1	square mile, sq. mile.
36	square miles	"	1	township, tp.

1 sq. m. = 640 a. = 6400 sq. ch. = 102400 p. = 64000000 sq. l.

12. Land Measure, French.

283. 100 centiares make 1 ARE (ar.) = 119.6 sq. yd.
100 ares " 1 hectare (ha.) = 2.471 acres.
A centiare is a square metre—or 1550 sq. in.

13. Square Measure.

284.

144	square inches (sq. in.)	make	1	square foot, sq. ft.
9	square feet	"	1	square yard, sq. yd.
30¼	square yards	"	1	square rod, sq. rd.
40	square rods	"	1	rood, r.
4	roods	"	1	acre, a.
640	acres	"	1	square mile, sq. m.

1 sq. m. = 640 a. = 2560 r. = 102400 sq. rd. or p. = 3097600 sq. yd.

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285. 14. Cubic Measure, English.

1728	cubic inches (cu. in.)	make	1 cubic foot,	cu. ft.
27	cubic feet	"	1 cubic yard,	cu. yd.
128	cubic feet	"	1 cord,	c.

15. Cubic Measure, French.

286. (Derived from French linear measure.)

1000 cu. mm. (mm^3) make 1 cu. cent. (cm^3) = .061 cu. in.
1000 cm^3 " 1 cu. decim. (d^3) = 61.027 cu. in.
1000 d^3 " 1 cu. metre (m^3) = 1.308 cu. yd.

16. Dry or Liquid Measure, French.

287. (The litre, or d^3, is the unit. It equals $\frac{33}{35}$ qt.)

1 millilitre	= $\frac{1}{1000}$	of a litre	= .0338 fluid oz.
1 centilitre	= $\frac{1}{100}$	"	= .353 oz. av.
1 decilitre	= $\frac{1}{10}$	"	= 3.527 "
1 litre	= 1	litre	= 2.2046 lbs.
1 decalitre	= 10	litres	= 22.046 "
1 hectolitre	= 100	litres	= 220.46 "

Ex. LXXXI.

17. Dry Measure, English.

288.

2 pints (pt.)	make	1 quart,	qt.
8 quarts	"	1 peck,	pk.
4 pecks	"	1 bushel,	bu.

The Eng. imperial bu. = 2218.2 cu. in.; the *standard U. S.* bu. 2150.4 cu. in.

18. Liquid Measure, English.

289.

4 gills (gi.)	make	1 pint,	pt.
2 pints	"	1 quart,	qt.
4 quarts	"	1 gallon,	gal.
31½ gallons,	"	1 barrel,	bbl.

NOTE.—2 bbls. are considered as 1 hogshead, hhd.; 2 hhds. as 1 pipe, p.; and 2 pipes as 1 tun, t.

There are three kinds of gallons, of which the liquid gallon

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contains 231 cu. in.; the dry gallon 268.8 cu. in.; and the imperial (Eng.), 277.27 cu. in.

290. *19. Circular Measure*

is used for noting differences in clock-time, in latitude, in longitude, and in the direction of diverging lines.

60 seconds (″)	make	1 minute,	′
60 minutes	"	1 degree,	°
360 degrees	"	1 circum.,	cir.

291. *20. Time Measure.*

60 seconds (sec.)	make	1 minute,	m.
60 minutes	"	1 hour,	h.
24 hours	"	1 day,	d.
365 days	"	1 common year,	c. y.
366 days	"	1 leap year,	l. y.

292. *21. Miscellaneous Table.*

12 units	= 1 dozen.	7 lbs.	= 1 clove (wool).
12 dozens	= 1 gross.	14 "	= 1 stone.
24 sheets	= 1 quire.	240 "	= 1 pack (wool).
20 quires	= 1 ream.	200 "	= 1 bbl. pork, beef.
2 reams	= 1 bundle.	196 "	= 1 " of flour.
5 bundles	= 1 bale.	4 inches	= 1 hand.
20 units	= 1 score.	18 "	= 1 cubit.
56 lbs.	= 1 firkin.	6 feet	= 1 fathom.
56 "	= 1 bu. rye, (N. Y.)	4 pp.	= 1 sheet folio.
60 "	= 1 " wheat, "	8 "	= 1 " quarto, 4to.
48 "	= 1 " barley, "	16 "	= 1 " octavo, 8vo.
48 "	= 1 " buckwheat, "	24 "	= 1 " 12mo.
58 "	= 1 " maize, "	32 "	= 1 " 16mo.
32 "	= 1 " oats, "	36 "	= 1 " 18mo.
56 "	= 1 " salt, "	48 "	= 1 " 24mo.
84 "	= 1 chest tea.	5 years	= 1 lustrum.

(For remarks on the tables see PART II.)

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Questions on Denominate Numbers, D. N.

293. NOTE.—Recitations on these questions should be required as the successive sets of problems are taken up.

What are D. N.? What is a Simple D. N.? A Compound D. N.? What question can *Number* answer? Are there any natural units by which we can measure the length of a river or the contents of a cask? What are signs or abbreviations used for? What are these signs generally? What does £ come from? What cwt.? What $? When are D. N. similar in kind? Mention similar D. N. Are gallons and bushels similar? What is a Standard Unit? What is it in U. S. money; in Land measure; in Time measure; in Cloth measure? What does a Table show? In writing a compound D. N., where should the lowest denomination stand? On what does the legal value of foreign money depend? Repeat the Table of U. S. money. State the value of an eagle in $, ds., cts., and ms. State the number of cts. in a half dollar, in a quarter dollar, in a third, a fifth, a sixth, an eighth, a tenth, a twentieth, and a twenty-fifth of a dollar. When the cost of one thing is an aliquot of a dollar, how is the cost of any number of like things found? Repeat the table of English currency. What are its abbreviations? The table of French currency. Its abbreviations. How much is a £ worth in our money? How much is a franc worth? Repeat the first three tables of weights. Give the signs of Troy weight; of Avoirdupois weight; of Apothecaries' weight. State the equivalent of a Troy lb.; of an Avoirdupois lb.; of a lb. Apothecaries; of a kilogram. State the names used in Linear measure and their signs. Give the scale of increase. State the equivalents of a mile in fur., rds., yds., and ft. What is the Unit of French linear measure? What are the Multiples? What are the Decimals? (Same questions respecting French weights.) How many inches in a metre? What is a centimetre? How many grains in a gram? How many grams in a kilogram or kilo? How many pounds avoirdupois in a kilo? What does

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this sign, viz., mm., mean? This, viz., dm., mean; dm., cm., hm.? What kind of letters make the first part of the signs of the multiples? Are the multiple words the same in every kind of French D. Ns.? Are the decimal words? What is the Unit of French money? *Ans.* The Franc. What are the units of length, capacity, weight, and area? How many cts. in a fr.; sq. yds in an are; lbs. in a kg.; cu. in. in a litre; inches in a metre? Write the equivalents of a kilometre, kl., kg., in each of the similar denominations. Write out the summary (verbatim) of the metric system. Assuming that English surveyor's linear measure is made up of links, chains, and miles, what part of a chain is squared to make a p.? What does a sq. ch. make? How many sq. ch. in an acre? What equals a sq. mile? What is a township? How many sq. metres in an are? What is a centiare in sq. metres? How many square inches in a sq. metre? ($1550 = 39\frac{3}{8} \times 39\frac{3}{8}$). Repeat the table of sq. measure, Eng.; of cubic measure, Eng.; of cubic measure, French; of capacity, French; of capacity, dry measure, Eng. How many cu. in. in a stand. bush., Eng.? How many in a stand. bush., U. S.? Liquid measure, Eng.? Liq. measure, French? How many cu. in. in a wine qt., U. S.; in a qt., Eng.; in a litre? (57.75, 69.82, 61.03). For what is circular measure used? What is an angle? etc., etc. Repeat the table. Ditto Time measure. Ditto Miscellaneous table.

REDUCTION OF CONCRETE NUMBERS.

294. Reduction is the process of **changing the form** of a number without changing its value.

NOTE.—In fractions the above definition was illustrated by the examples in which improper fractions were found equal to integers, and by those in which simple fractions were found equivalent to compound fractions.

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295. The **reduction** of a **denominate number** consists in finding how many units of one kind are contained in any given number of another kind.

Let it be required to find how many ounces there are in 10 lbs. As in 1 lb. there are 16 ounces, in 10 lbs. there are 10 times 16 ounces, or 160 ounces.

Ex. 2. How many ounces in 2 t. 2 cwt. 2 qr. 2 lbs. 2 oz.?

	t.	cwt.	qr.	lbs.	oz.
(*b*)	2	2	2	2	2
		20			
(*a*) 20 cwt. × 2 + 2 cwt. = 42 cwt.		42	cwt.	(2 being added.)	
		4			
4 qr. × 42 + 2 qr. = 170 qr.		170	qr.	"	
		25			
25 lbs. × 170 + 2 lbs. = 4252 lbs.		4252	lbs.	"	
		16			
16 oz. × 4252 + 2 oz. = 68034 oz.		68034	oz.	"	

It will be seen that *Ex.* 2 has two solutions, *a* and *b*. As a multiplier is always an abstract number, and as a product is like the multiplicand, it would appear that of the two solutions only *a* is theoretically sound. But as in fact merely changing the places of factors does not alter a product, solution *b* leads to the same result; and, as it is the briefer, it is to be preferred. We might perhaps reason thus—taking *Ex.* 1 as an instance, viz.: "How many ounces in 10 lbs? It would appear that there are 16 times as many ounces in any package as there are pounds; and as there are 10 lbs., there are 16 times 10 ounces. We regard the quantities *abstractly* while thus reasoning." If this reasoning be readily followed by a class, the following may be taught as a *rule* for reducing a compound number to a lower denomination:

296. "*Reduction Descending.*"

RULE.—*Multiply the number of the highest denomination in the given quantity, by the number of units in the next lower*

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denomination contained in a unit of the highest; and to the product add such as there are of the second denomination. Reduce this result to the next lower denomination in the same way, and continue till the denomination required is reached.

Exercise LXVII. (Table 2).

1. Reduce £5 2s. 6d. to pence; to halfpence; to far.
2. Reduce £3 3s. 3d. to sixpences; to halfpence; to far.
3. How many books at 1s. 6d. each will £150 purchase?
4. How many apples at 2½d. each will £5 5s. 5d. buy?
5. How many melons at 2s. 2d. for £9 18s. 10d.?
6. How many sixpences in £40?
7. How many farthings in £27, 50 half-crowns, 87 shillings, and 43 sixpences, taken together?
8. In £58 12s. 6d. how many francs, at 10d. each?
9. In £20 how many dollars, at 4s. 2d. each?

Exercise LXVIII. (Table 3).

1. Reduce 10000 francs to dollars. (Divide by 5.18.)
2, 3. 120000 centimes to dollars; 1250 francs to dollars.
4–6. 7340 francs to centimes; 12000 cent. to fr.; to $.
7–9. $1850 to centimes; $291.60 to fr.; 1964 fr. to cent.
10. Allowing $4.86¾ to the £, and 5.18 fr. to the $, how many fr. and cent. in £1? *Ans.* 25 fr. 22.3 cent.
11, 12. In £24 (605.36)? In £14 6s. 8d. (£14⅓)?
13–16. In $42? $68? $175.50? $34.65? $850? $7564.56?
17–20. How many dollars in 1200 fr.? 20000 fr.? 1864.15 fr.?

Exercise LXIX. (Table 4).

Reduce each of the following to the denomination required:
1–6. 4, 7, 9, 13, 172, 454 dwts. to grains.
7–11. 3, 19, 26, 148, 396 ounces to grains.

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12–16. 2, 12, 13, 26, 280 lbs. to dwts.

17, 18. 2 lbs. 2 dwts.; 4 oz. 4 dwts. 4 grs. to grs.

19, 20. 5 lbs. 5 oz. 5 dwts.; 3 lbs. 4 oz. 5 dwts. to dwts.

21–24. How many grs. in 5 lbs.? 4 oz.? 7 dwts.? 19 oz.?

25, 26. How many ounces in 168 lbs.? 2000 lbs.?

27. Req. the quotient of 15 lbs. 8 oz. 5 dwts. 15 gr. divided by 2 lbs. 1 oz. 2 dwts. 2 grs. (Reduce both to grs.) *Ans.* $7\frac{1}{2}$.

28. Divide 2 lbs. 2 oz. 14 dwts. 16 grs. by 3 oz. 4 dwts. 4 grs. *Ans.* $8\frac{128}{385}$.

29. Reduce 68 lbs. 2 oz. 6 dwts. 20 grs. to grains.

Ans. 360164 grs.

EXERCISE LXX. (TABLE 5).

1–3. Req. no. of qrs. in 16 cwt. In 24? In 127?

4–7. No. of ozs. in 14 lbs.? In 26? In 37? In 100?

8–11. No. of drs. in 28 lbs.? In 52? In 74? In 50?

12–14. No. of lbs. in $5\frac{1}{2}$ t.? In $21\frac{1}{8}$ t.? In $7\frac{1}{40}$ t.?

15. Reduce to drs. 1 t. 1 cwt. 1 qr. 1 lb. 1 oz.

16. Reduce 20 t. 13 cwts. 3 qrs. 8 lbs. 10 oz. 1 dr. to drams.

Ans. 10594209 drs.

17–19. Reduce to drs. 57 lbs. 14 oz.; 16 lbs. 9 drs.; 1 t. 2 cwts.

20, 21. Reduce 2 t. 2 cwt. 2 qrs. 2 lbs. to ozs.; $5\frac{1}{4}$ cwts. to ozs.

22, 23. Reduce to qrs. 4 t. 9 cwt. 12 qrs.; 186 t. 4 cwt.

24. Reduce to drams 10 t. 6 cwts. 3 qrs. 16 lbs. 13 oz. 1 dr.

Ans. 5297105 drs.

EXERCISE LXXI. (TABLE 6).

How many drams in each of the following:

1–7. 13 oz.? 7 oz.? 19 oz.? 162 oz.? 1425 oz.? 2 lb. 2 oz.? 2 lbs. 1 oz.?

8–10. How many scruples in 1 lb. 2 oz. 4 scru.? In 42 oz.?

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11–13. In 1 lb. 1 oz. 1 dr.; in 14 lbs. 2 drams; and in 16 oz. 7 scru., taken together, how many drams?

14–18. How many grains in 12 scru.? In 16 drs.? In 4 lbs.? In 1000 lbs.? In $\frac{1}{2}$ of $\frac{2}{3}$ of 36 lbs.

19, 20. How many grains in 5 dr. 2 scru.? In 2 oz. 2 dr.?

21. Reduce 69 lbs. 11 ℥ 3 ʒ 2 ℈ 8 grs. to grains.

Ans. 402948.

22–25. Reduce to scruples 97 lbs. 2 oz.; 142 lbs. 5 oz. 5 drs. 5 ℈; 13 lbs. 6 ℈; 11 oz. 5 drams 2 scru.

26. Reduce to drams $\frac{7}{1000}$ of a lb. avoirdupois (1 lb. Troy).

27. Reduce 3500 lbs. avoirdupois to lbs. apoth.

28–31. 1800 lbs. Troy; 2560 lbs. Troy; 240 lbs. Troy; and 168 lbs. Troy to lbs. avoirdupois.

Exercise LXXII. (Table 7).

1–3. Reduce to grams 2 decagrams; 12 deca.; 18 deca.

4–6. Reduce to grams 12 hectogr.; 2 kilog.; 2 myriagr.

7, 8. Reduce to kilos. 5 quintals; 12 steres or milliers.

9. Reduce to kilos. 7 milliers 7 quintals 7 kilos.

10. Reduce 96 kilos. (each 2 lbs. $3\frac{1}{4}$ oz.) to drams avoird.

11. If 9 tons 16 cwt. 3 qrs. 12 lbs.=1 millier, how many lbs. in 6 millier?

12. A kilo. = 2 lbs. 8 oz. 3 dwt. 2 grs. Troy. How many grains in 4 kilos.; in 30 hectogr.; in 126 decagrams?

13. A hectogr.= 3.527 oz. avoirdupois. How many lbs.= 9746 hectogr.?

14, 15. 453.6 grams = a lb. avoird. How many grains in 1000000 grams? In 1000 kilograms?

16–20. 1 quintal = 220.5 lbs. avoird. How many drams in 16 qu.; in 37; in 116; in 224; in 990?

Exercise LXXIII. (Table 8).

1. Write the Table for English Linear Measure complete.

2–4. How many miles in 4 degrees? In 62? In 17?

5–7. How many rods in 22 mi.? In 100? In 119?

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8–10. How many inches in 4 ft.? In 2 yds.? In 10?
11, 12. How many inches in $5\frac{1}{2}$ yds.? In 3 rods?
13. How many feet in 1 mi. 6 fur. 8 rds. 2 yds.?
14. Reduce 2 mi. 3 fur. 4 rods 5 yds. 2 ft. to inches.
15. Reduce 1 mi. 2 fur. 3 rods 4 yds. 5 ft. to inches.
16, 17. Reduce 5 mi. 3 fur. 2 yds. to feet; 2 mi. 2 fur.
18–20. Reduce to inches $\frac{3}{4}$ of a mi.; $\frac{4}{5}$ of a fur.; 25 rods.

EXERCISE LXXIV. (TABLE 9).

1. Write out Table 9. Multiply it by 12.
2. How many inches in 12 decimetres?
3. How many decimetres in 133 decametres?
4. A metre is 3.281 ft. How many ft. in a myriametre?
5. How many inches in 10000 millimetres?
6. How many metres in a million millimetres?
7. How many inches in 1111.111 metres?
8. How many rods in 1764 hectometres?
9. How many miles in 42693 decametres?
10. How many miles in 10,000,000 metres?

EXERCISE LXXV. (TABLE 10).

1. Write the Table. How many in. in a mi.?
2, 3. How many links in 40 ch.? In 16 ch.?
4, 5. How many links in 100 rods? In 75?
6, 7. How many inches in 792 links? In 1120?
8. How many inches in 1 mi. 1 ch. 1 rod 1 link?
9, 10. How many ch. in 42 mi.? In 976 miles?
11. Req. the difference, in links, between 13 ch. and 11 ch.?
12. If 66 ft. = 1 ch., how many feet in 7296 ch.?
13, 14. How many rods in 2 mi. 48 ch.? 1 mi. 26 ch.?
15. Reduce 4 mi. 4 ch. 4 rods 4 links to inches?

EXERCISE LXXVI. (TABLE 11).

The area of a rectangle = length × width.
1. A field is 40 ch. long and 16 ch. wide; how many a.

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2, 3. Req. a. in a field 90 ch. by 72 ch.; 100 ch. by 100.

4. Req. sq. mi. in a township 1000 ch. by 500 ch.

5. Req. sq. in. in a plot 45 links by 60.

6. Req. area in sq. yards of a plot 60 rods by 40.

7. Req. number of a. in a field 82 by 71 chains.

8. At 40 dollars an acre what will 1063125 sq. links of land cost? *Ans.* $425.25.

9. How many sq. chains in 275.4 a.? *Ans.* 2754.

10. In 97 sq. miles how many sq. ch.?

Exercise LXXVII. (Table 12).

1, 2. How many sq. metres in 2 hectares? 4 hectares?

3. 1 are = 119.6 sq. yds. Sq. ft. in 211 ares?

4–6. How many sq. yds in 20 ares; in 72 ares; in 91½ ares?

7. How many sq. yds. in 42 hectares?

8. How many centares in 11 hectares?

9, 10. How many ares in 116 acres? In 45?

11–13. How many centares in 5 hectares 1 are? In 100 hectares 1 are? In 122 hectares 4 ares?

14, 15. How many acres in 17 hectares? In 23?

16, 17. How many sq. rods in 96 hectares? In 102?

18. How many sq. ft. in 9642 centares?

19, 20. How many sq. in. in 4 centares? In 29?

Exercise LXXVIII. (Table 13).

1, 2. How many sq. yds. in 2 sq. mi.? In 7?

3, 4. " " feet in 5 " In 31?

5, 6. " " rds. in 72 " In 47?

7, 8. " " in. in 3 sq. rods? In 116?

9. At $10 a sq. rod what cost 4 acres?

10. At 6 cts. a sq. yd., what cost 1 acre?

11. At 2 cts. a sq. ft., what cost 5 sq. miles?

12. In 2 sq. mi. 5 roods 3 sq. rods how many sq. rods?

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13. What is the number of ares in a sq. mi.?

14. How many hectares in 759 sq. mi.?

EXERCISE LXXIX. (TABLE 14).

297. The volume of a solid is found by taking the continued product of its length, breadth, and thickness.

1, 2. Find the product of $12\times12\times12$; of $6\times6\times6$.

3, 4. " " $3\times3\times3$; of $9\times9\times9$.

5. Req. cu. in. in a box 7 ft. by 8 ft. by 9 ft.

6–10. How many cu. in. in 186 cu. ft.? How many in a stere? In 712 cu. ft.? In 2 cu. yds.? In 7 cu. yds.?

11. A cu. ft. of water weighs 997 oz. (commonly stated, 1000 oz.); how much will a gallon of water weigh?

12. A barrel of water contains 231 cu. inches $\times$ 31.5; what will it weigh? (1728 cu. in. of water weigh 1000 oz.)

13, 14. In digging a cellar 25 by 40 by 8 ft. how many cu. yds. of earth must be moved? How many cu. metres?

15, 16. How many cu. inches in a block 14 in. by 10 in. by 8 in.? In one $110\times100\times62$?

EXERCISE LXXX. (TABLE 15).

NOTE.—The unit of large masses is the *stere.*

1. How many millisteres in a stere?

2. How many decisteres in a decastere?

3. A stere = 35.317 cu. ft. How many cu. in. in a decastere?

4. A stere = a cubic metre; a litre is a cubic decimetre, or 61. cubic inches. How many litres in a stere?

5. How many centilitres in a stere or myrialitre?

6–8. How many gallons of 268.8 in dry measure in a myrialitre? In a hectolitre? In 46 hectolitres?

9. How many litres in 4632 bushels? (2150.4 in. = 1 bush.)

10, 11. How many bushels in 10000 litres? In 21240 litres?

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12. If a litre of water is a kilogram in weight, or $2\frac{1}{6}$ lbs., how heavy is a millier or stere of water? 2204.6 lbs.

EXERCISE LXXXI. (TABLE 16).

298. NOTE.—A litre = .908 of a dry quart or 1.057 of a liquid quart.

Names:	Decilitre,	*litre,*	decalitre,
Values, dry measure:	.0908 qts.	.908 qts.	9.08
" liquid "	.10567 "	1.0567 "	10.567

Names:	hectolitre,	and cubic metre.
Values, dry measure:	90.8	908. qts. or $28\frac{3}{8}$ bush.
" liquid "	105.67	1056.7 or 264.17 gals.

1, 2. How many litres in 25 bushels? In 32 bushels?

3, 4. How many litres in 64 gals. wine? In 908 gals.?

5, 6. How many bbls. wine in 10000 litres? In 2468?

7–10. How many bushels in 60 hectol.? In 100? In 984? In 76?

EXERCISE LXXXII. (TABLE 17).

1. Req. value, at \$1.20 per bushel, of a bin of grain measuring 46 in. by 38 in. by 36 in.

2. Req. val., at \$2.40 per bushel, of contents of a bin 22 ft. by 20 ft. by 8 ft. (l. × br. × dep. × 1728 ÷ 2150.4 × \$2.40).

3. Req. value of 96 bushels of seed at 25 cts. per gallon.

4. Req. value of 20 cubic yards of wheat at \$1.20 a bushel?

5. How many cu. inches in 24 bushels?

6, 7. How many pints in 18 cu. ft.? In 21 cu. yds.?

8. How many gals. in 963 bushels?

9–11. A vat 8 ft. by 6 ft. by 10 in. will hold how many bushels; how many gals.; how many pints?

12. A bushel of wheat weighing 60 lbs., how many lbs. will a vat 40 by 20 by 10 ft. contain?

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Exercise LXXXIII. (Table 18).

1–3. Reduce 2 gals. to gills; also 8 gals.; 9 gals.

4–6. Reduce 3 bbls. to gals.; 12 bbls.; 13 bbls.

7–11. Reduce 1 hhd. 1 bbl. 1 gal. 1 qt. and 1 pt. to gills.

12–14. Reduce 506 hhds. to gals.; 719 hhds.; 416 hhds.

15. Req. value of 21 bbls. wine at $2.50 per gallon.

16. Bought 4 hhds. of molasses at $30 a hhd., and sold it at 32 cts. a qt.; what was gained?

17. Bought 3 bbls. of oil at 75 cents a gallon, and sold it all for $120; required the gain.

18, 19. If 54 gals. fill a hhd. of beer, how many pints in 75 hhds.? In 100 hhds.?

20. If 282 cu. in. make a beer gallon, how many cu. inches in 400 gallons?

299. As the earth requires 24 hrs. or 1440′ to make a complete revolution under the noonday sun, every degree of longitude makes a difference of $\frac{1440}{360}$ minutes (or 4 minutes) in clock time *later* toward the west, or *earlier* if the place at which the time is required lies easterly.

To find the difference in Time, when the difference in Longitude is given.

Rule.—*Divide the difference in longitude by* 15 *and the result, in degrees, minutes, and seconds, will represent hours, minutes, and seconds. Or, reduce the difference in longitude to degrees and multiply by* 4. *The product will represent minutes of time.*

Exercise LXXXIV. (Table 19).

1–3. How many minutes in 70°? In 91°? In 4°?

4. How many minutes in $\frac{1}{10}$ of a right angle or 90°?

5. How many minutes in 6 S. 27° 43′? (S.=30°).

6, 7. How many seconds in 360°? In 160°?

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8. Allowing 4 minutes of time to a degree, what difference in time does 30° make?

9, 10. Boston is 71° 4′ west of Greenwich; San Francisco is 51° 17′ west of Boston: what is the time at Greenwich, and what at San Francisco, when it is noon in Boston?

(G.) 4 h. 44′.16″ P. M.
(San F.) 8 h. 34′.52″ A. M.

Exercise LXXXV. (Table 20).

1–3. How many hours in 10 days? In 24 days? In 1 year?

4–6. How many minutes in 2 days; 3 days; 2 weeks?

7, 8. In 4 days how many seconds? In 58 hours?

9. Reduce 2 w. 2 d. 2 h. 2 m. 2 sec. to secs.

10. Reduce 4 years to minutes ($365\frac{1}{4}$ days in a year).

11. Reduce 365 d. 5 h. 48 m. $49\frac{7}{10}$ sec. to seconds.

12. How many weeks in 110 years ($52\frac{5}{28}$ w. in 1 yr.)?

13, 14. How many seconds in 10 weeks? 10 months of 30 days?

15. How many days in 49 years 3 months?

Exercise LXXXVI. (Table 21).

1. How many units in 2 doz. 2 score and 2 gross?

2. How many sheets in 4 reams and 12 quires?

3. How many sheets in 75 reams and 18 quires?

4. How many lbs. in 1 bush. of rye, 1 of wheat, 1 of barley, 1 of buckwheat, and 1 of maize, taken together?

5. How many lbs. in 30 bbls. pork and 40 bbls. flour?

6. A man weighs 12 stone 4 lbs.; how heavy is he?

7, 8. How many lbs. in 1142 stone? In 176?

9, 10. How many inches in 17 hands? In 15?

11, 12. How many feet in 900 fathoms? In 2264?

13, 14. How many are 8 dozen dozen? 6 dozen dozen?

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15. If a tailor takes 14 doz. and four buttons from a package of 5 gross, how many will remain?

16, 17. How many sheets in 7 bundles? In 9?

18. How many quires in 4 bales?

19. If each of 700 books require 14 sheets, how many reams will be needed?

20. If a workman makes 20 dozen books in a day, how many will he make in 304 days?

21. Each of 18 boys has 5 boxes of pens: if there be a gross in each box, how many pens in all?

22. If 12 reams of paper be bought at $2.75 per ream, and it is sold at a cent a sheet, what is the entire profit?

23–25. When paper is $5 a ream, what is the cost of a quire? At $7 a ream? At $9?

26. At $1\frac{1}{2}$ cents for 5 sheets what will 15 quires of paper cost?

27. If one book contain 16 sheets, how many bundles must be had to make 4000 books?

NOTE.—The problems which have been given in the last 27 exercises will have fully exemplified what is commonly termed "Reduction Ascending," *provided that* **the pupils have been required to prove their work.** The rules are obviously complements of each other.

Reduction Ascending.

300. Reduction Ascending is the process of finding the value of a number expressing **small** units in terms of another number expressing **larger** units.

Example 1. Let it be required to reduce 68034 ounces to a compound D. N. of larger units.

16)68034
25)4252 lbs. and 2 oz.
4)170 qrs. and 2 lbs.
20)42 cwts. and 2 qrs.
2 t. and 2 cwts.

Explanation.—There is one-sixteenth as many lbs. as there are oz., and one-twenty-fifth as many qrs. as there are lbs. So there is one-fourth as many cwts. as there are qrs., and one-twentieth as many tons as cwts. The

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last quotient and the several remainders will equal the compound D. N., 2 t. 2 cwt. 2 qr. 2 lbs. 2 oz.

To reduce a Denominate Number to higher Denominations.

Rule.—*Divide the given number by as many of its kind as will make one of the next higher. Set apart the remainder, and divide the first quotient by as many of its kind as make one of the next above it. Continue to divide each new quotient until the highest is reached. The last quotient with the several remainders will form the equivalent compound number.*

Make an exercise of five problems under each table.

301. *Addition of Denominate Numbers.*

£	s.	d.	qr.
7	11	5	1
4	9	6	2
5	8	7	1
3	12	0	3
19	40	18	7
21	1	7	3

In practice, the second footing only is set down. The sum of the qrs., 7, equals 1d. and 3qrs. After writing these 3qrs., the 1d. is added to the column of pence, the sum of which, viz., 19d., is considered as 1s. and 7d. The 1s. is carried into the column of shillings, and the sum, 41s., is considered as £2 and 1s. The £2 added to the column of pounds makes £21. The remainders after each reduction are set under their respective columns.

302. Rule for the Addition of Compound Numbers.—*Place the numbers so that the same denominations may stand under each other. Add the several columns separately, beginning with the lowest, and divide the sum of each by the number of its kind required to make one of the next higher name; set down the remainder and add the quotient to the next column. Prove by re-addition, beginning at the top.*

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EXERCISE LXXXVII.

303. Consider the following numbers as representing the **different denominations,** beginning at the lowest **of each of the following measures,** viz., Linear, Square, Land, Circular, and Liquid, also of Avoirdupois and Apothecaries' Weight—21 problems.

1–7.	20	3	4	6	1–7.	10	6	4	2	1–7.	8	4	5
	176	2	9	7		12	3	5	1		6	9	3
	13	14	3	2		9	4	2	2		8	2	1
	17	6	5	1		17	3	5	4		4	1	3
	126	2	4	3		11	2	4	1		8	2	5

304. *Subtraction of Denominate Numbers.*

Ex. 1. From £24 11s. 8d. 3qr. take £18 15s. 5d. 1qr.

The learner will have observed that 15s. cannot be taken from 11s. Of the £24, £1 was taken and reduced to shillings and added to 11s.; then from the sum 15s. was taken, leaving 16s. £18 from £23 leaves £5. Pupils may be taught as follows: If £1 be added to each quantity, the difference will still be the same. Adding 20s. to 11s., from the sum, 31s., 15s. may be taken. Adding 20s. in the form of £1 to £18, and subtracting the sum, £19, there remains £5, as before. See rule for simple Subtraction.

	£	s.	d.	qr.
	24	11	8	3
	18	15	5	1
	5	16	3	2

Proof: 24 11 8 3

305. RULE FOR THE SUBTRACTION OF COMPOUND NUMBERS. —*Write the less number under the greater, so that each number may stand under one of the same name. Beginning with the lowest name, take the lower number from the upper. If the lower is the greater, add to the upper as many as will make one of the next higher denomination, and subtract; add one*

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to the next higher number of the subtrahend before subtracting it in its turn.

PROOF.—*Add the remainder to the less number.*

EXERCISE LXXXVIII.

1. From £2 3s. 4d. take £1 2s. 9d.
2. From £7 8s. 9d. take £6 9s. 8d.
3. From £7 take 5 guineas.
4. From 3fr. 3dec. 2c. take 1fr. 1 dec. 1c. (332–111).
5. From 674fr. take 3fr. 2d. 1 cent. (decimal).
6. From 125 lbs. 4 oz. 16 dwt. take 11 lbs. 3 dwt.
7. From 48 lbs. 17 grs. take 14 oz. 15 dwt.
8. From 4 cwt. 1 qr. 3 lbs. take 2 qrs. 20 lbs.
9. From 16 lbs. 5 oz. 3 scr. take 10 lbs. 10 drams.
10. From 43 miles 22 poles take 3 fur.
11. From 15 y. 2 ft. 3 in. take 7 y. 8 in.
12. Take 365 d. 5 h. 48 m. 50 sec. from 365 d. 6 h. 9 m. 11 sec., viz., take a solar from a sidereal year.
13. Take 32 a. 3 r. from 800 a. 1 rood 16 perches.
14. Take 270 sq. yds. 4 sq. ft. from 287 sq. yds. 2 sq. ft.
15. Take 18 cords 8 ft. 1119 in. from 20 cords.
16. Take 3 hhds. 2 gals. from 4 hhds. 2 pts.
17. From 173 bu. take 124 bu. 2 pks. 1 qt.
18. From 23° 45′ 54″ take 12° 51′ 25″.
19. If a person buy a quantity of wheat for £1893 $\frac{3}{4}$d., and sell the same for £2000, how much will he make by the sale?

Ans. £106 19s. 11$\frac{1}{4}$d.

20. A silversmith had 34 pounds 9 ounces 10 pennyweights of silver; he melts 19 pounds 15 pennyweights 10 grains: how much has he left? *Ans.* 15 lb. 8 oz. 14 pwt. 14 gr.
21. If out of 6 pounds 10 ounces 6 drams 2 scruples of medicine be taken 4 pounds 5 ounces 4 drams 1 scruple 17 grains, what quantity will remain? *Ans.* 2 lb. 5 ℥ 2 ʒ 0 ℈ 3 gr.
22. Bought 145 yards 3 quarters of cloth, and sold thereof 95 yards 2 quarters 3 nails: how much remains?

Ans. 50 yds. 1 na.

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23. A farmer had 500 acres 1 rood of land; one of his sons having married, he gave him 150 acres 3 roods 25 poles; to another son he gave 24 acres 3 roods 5 poles; to each of his three daughters he gave 17 acres 1 rood and 17 poles: how much had he remaining?

24. Bought several casks of cider, containing 154 gallons 2 quarts, and disposed of one which contained 41 gallons 2 quarts 1 pint; how much is there in the other casks?

Ans. 112 gals. 3 qts. 1 pt.

25. Out of a granary which contained 283 bushels of corn, there were taken 153 bushels 2 pecks 5 quarts: what quantity remains? *Ans.* 129 bu. 1 pk. 3 qts.

26. William was bound as an apprentice for 7 years; he has served 3 years and 5 months: how long has he still to serve? *Ans.* 3 yr. 7 mo.

306. RULE FOR FINDING THE DIFFERENCE IN DAYS BETWEEN TWO DATES.—*Add the number of days in each intervening month to those in the two parts of months. Omit the day on which a note is dated, but add in the day on which it falls due.* (For long periods, see 307, Note.)

EXERCISE LXXXIX.

Ex. 1. A note dated July 10 is to be paid Dec. 4. Days?

July. Aug. Sept. Oct. Nov. Dec.

$21 + 31 + 30 + 31 + 30 + 4 = 147$ days.

2. Sept. 21, 1871, to Feb. 11, 1872. Days?

3. Dec. 12, 1872, to June 10, 1873. Days?

4. June 4, 1872, to Mar. 9, 1873. Days?

5, 6. Oct. 12 to Dec. 30? Jan. 10 to August 29?

307. NOTE.—*In finding the difference between two dates,* instead of the name of a month, express its number. Thus, write 2 for Feb., 7 for July, etc. (See next problem.)

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7. Washington was born on the 22d February, 1732, and died on the 14th of December, 1799. What was his age?

8. De Witt Clinton was born on March 2, 1769, and died on the 11th of February, 1828. At what age?

9. How many years, months, and days between Sept. 7, 1868, and Oct. 4, 1872?

308. *Multiplication of Compound Numbers.*

Multiply 17 lbs. 5 oz. 12 dwts. 6 grs. by 3.

A brief inspection of this operation will lead readily to the following:

17 lbs.	5 oz.	12 dwts.	6 grs.
			3
52 lbs.	4 oz.	16 dwts.	18 grs.

RULE FOR THE MULTIPLICATION OF A COMPOUND NUMBER. —*Multiply as in simple numbers and carry as in Addition of Compound Numbers.*

309. NOTE.—If the multiplier be a composite number, its factors may be used successively as multipliers. Thus, to multiply by 168, first multiply by 4, then multiply its product by 7, and this latter product by 6.

EXERCISE XC.

Multiply the given numbers as directed:

1–5. £10 15s. 6d. separately by 2, 5, 8, 11, and 49.

6–10. 17 lbs. 5 oz. 12 dwt. 6 gr. by 3, 7, 9, 12, and 29.

11–15. 5 tons 16 cwt. 2 qr. 12 lb. by 4, 5, 11, 15, and 21.

16–20. 14 yds. 1 ft. 10 in. by 7, 8, 11, 13, and 24.

21–25. 2 bush. 2 pks. 6 qts. by 3, 4, 5, 7, and 16.

26–30. 30° 15′ 12″ by 4, 9, 13, 18, and 25.

31–35. 35 lbs. 9 ℥ 4 ʒ 2 ℈ 15 gr. by 2, 3, 5, 7, and 9.

36–40. 15 a. 3 r. 12 p. by 7, 11, 12, 16, and 20.

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310. *Division of Denominate Numbers.*

10)42 lbs.	3 oz.	6 dwts.	
4 lbs.	2 oz.	14 dwts.	$14\frac{2}{5}$ grs.

Analysis.—The parts of the dividend are considered separately. $\frac{1}{10}$ of 42 lbs. is 4 lbs. The remainder is 2 lbs. This remainder, reduced to ounces and increased by 3 oz. equals 27 oz.; $\frac{1}{10}$ of which is 2 oz. The remainder, 7 oz., reduced to dwts. and increased by 6 dwts., equals 146 dwts.; $\frac{1}{10}$ of which is 14 dwts. The remainder, 6 dwts., reduced to grains and divided by 10, equals $14\frac{2}{5}$ grains.

RULE.—*Divide as in simple numbers. Reduce each remainder to the next lower name and add to it that part of the dividend which is of the same name, and divide as before.*

PROOF.—*Multiply the quotient by the divisor.*

EXERCISE XCI.

Divide the following by 4, 6, and 11 separately:

1–20. 13 acres; 15 lbs. Troy; 19 lbs. avoirdupois; £32; 45 francs; 17 miles; 47 kilometres; 39 chains; 29 hectares; 45 fathoms; 79 cu. yds.; 7°; 13 weeks; 25 steres; 509 litres; 87 bush.; 91 wine gals.; 17 metres; 71 kilograms; 51 quires. (20 groups of problems, 60 in all.)

THE RELATIONS OF NUMBERS.

311. The solution of most arithmetical questions turns on the relation between the numbers employed. *Any number may be regarded as a part of another number.* The result of the comparison of two numbers is, at first, a **fraction.** One number is written as the denominator, because it is the **stan-**

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dard, or the measure, and therefore the divisor; that which is considered as a **part** is written as the numerator, as its comparative **value** is to be determined or **measured.**

Exercise XCII.

1–21. What part of 2 is 1? Of 3 is 1? Of 4 is 1? Of 6 is 1? Of 10 is 1? What part of 4 is 2? Of 6 is 2? Of 8 is 2? What part of 6 is 3? Of 9 is 3? Of 12 is 3? What part of 12 is 6? Of 12 is 4? Of 12 is 3? Is 2? 2 is what part of 12? 3 is what part of 12? 4 is what part of 12? 6 is what part of 12? 1 is what part of 2? 2 is what part of 4?

312. It will be seen that the complete number is made the measure of the *part.* In Art. 133 it was shown that a measure and a divisor are the same. The part is to be divided by the number of which it is a part. **Of** *stands before the* **divisor** *or denominator, and the* **part** *is to be written as a* **numerator.**

Note.—When numbers are to be compared, they must be similar in kind or they must have a common unit. Thus, to ask what part of 4 quarts is 1 quart is the same as to ask what part of a gallon is one quart. A gallon may be considered as part of a cu. ft., as it consists of 231 cu. in., and the cu. ft. contains 1728 cu. in.

Exercise XCIII.

1. **Six problems analyzed.** What part of 2 yards is 5 feet? 2 yards equal 6 feet: as 1 foot is 1 sixth of 6 feet, 5 feet is 5 sixths of 6 feet or of 2 yards.

2. What part of 1s. 6d. is 8d.? 1s. 6d. equals 18d.; 8d. is 8 eighteenths of 18d. ∴ 8d. is $\frac{8}{18}$ of 1s. 6d.

3. What part of 1 lb. is 2 oz. 2 dwts. 2 grs.? 2 oz. 2 dwts. 2 grs. equals 1010 grs. In 1 lb. there are 5760 grains; 1010 grs. is $\frac{1010}{5760}$ of 5760 grs.

4. What part of 1 yard is 2 feet 3 inches? 1 yard equals 36

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inches; 2 ft. 3 in. equal 27 inches; 27 in. is 27 thirty-sixths of 36 inches or of 1 yard.

5. What part of a foot is $\frac{3}{8}$ of an inch? 1 inch is $\frac{1}{12}$ of 1 foot; $\frac{3}{8}$ of an inch is $\frac{3}{8}$ of $\frac{1}{12}$ or $\frac{3}{96}$ of a foot. $\therefore \frac{3}{8}$ in. $= \frac{3}{96}$ ft.

6. What part of 1 lb. is $\frac{2}{7}$ of a dram? 1 dram is $\frac{1}{16}$ of an oz. or $\frac{1}{16}$ of $\frac{1}{16}$ of a lb., or $\frac{1}{256}$ of a lb. $\frac{2}{7}$ of a dram is $\frac{2}{7}$ as great a fraction as a whole dram, and $\frac{2}{7}$ of $\frac{1}{256}$ lb. equals $\frac{2}{1792}$ of a lb.

313. From these analyses, if they have been fully understood, it will have been seen that when denominate numbers, either simple or complex, are compared, they must be reduced to the same name. If the part be a fraction, and it is compared with a unit of a higher name, it may be divided by multiplying its denominator by the number of units of its own name required to make one of the kind of which it is said to be a part. (See Division of a Fraction, Examples 5 and 6.) If a fraction of a higher name is to be reduced to a fraction of a lower name, it must be multiplied by the same scale numbers required in reducing an integral number. Thus, to reduce $\frac{3}{100}$ of a bu. to the fraction of a qt., $\frac{3}{100}$ must be multiplied by 4 times 8; the result is $\frac{96}{100}$ qts.

314. To reduce a Fraction to one of a higher Denomination.

Rule.—*Multiply the denominator by the number of the given name which is required to make one of the denomination sought, and write the product under the original numerator.*

315. To find what Compound Number will equal a given Fraction.

Rule.—*Multiply the numerator of the given fraction by the number which will reduce it to the next lower name, and divide by its denominator. If there is a remainder, multiply it by the number of the next lower which makes one of its name, and divide as before. The several quotients will be the compound number required.*

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316. To reduce a Compound Number to one of a higher name.

Rule.—*Reduce the compound number to its lowest name for a numerator, and the higher unit to the same name for a denominator.*

317. To find what Decimal Fraction one Concrete Number is of another.

Rule.—*First find the common fraction by one of the above methods, and reduce it to a decimal.*

Exercise XCIV.

Reduce the following denominate fractions as required:

1–23. $\frac{4}{5}$ of an hour to the fraction of a day, of a week, of a year; $\frac{5}{9}$ of a foot to the fraction of a yard, of a rod, of a furlong, of an inch; $\frac{2}{11}$ of a gallon to the fraction of a pint, of a hhd., of a quart; $\frac{5}{6}$ of a rod to lower names [4 yds. 1 ft. 9 in.]; $\frac{5}{9}$ of a mile to lower names [4 fur. 17 rds. 4 yds. 10 in.]; $\frac{1}{7}\frac{1}{2}\frac{1}{0}$ of a bushel to pints. What fraction of a sq. ft. is $\frac{9}{484000}$ of an acre? What part of an in. is $\frac{1}{100000}$ of a mile? What part of 1 lb. is $\frac{2}{5}$ dwt? $\frac{3}{5}$ oz.? What part of a circle is 1°; is 5′; $\frac{2}{7}$°; $\frac{5}{13}$′; 25″; $\frac{24}{40}$″?

Exercise XCV.

Reduce the following:

1–10. 1 lb. 1 oz. 1 dr. to the fraction of a cwt. 1 oz. 1 dwt. 1 gr. to the fraction of a lb. 2 oz. 3 dwt. 4 gr. to the fraction of a lb. 1 rood 2 p. 1 sq. yard to the fraction of an acre. 6 fur. 16 rods to the fraction of a mile ($\frac{4}{5}$). 3 pks. 4 qts. to the fraction of a bush. 2 qts. 1 pt. 2 gills to the fraction of a gallon. 5 days 11 hours to the fraction of a week. 4s. 3d. 2qr. to the fraction of a £. 2 oz. 4 dr. 5 scru. to the fraction of a lb.

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318. To reduce a Denominate Number to the decimal of a higher denomination.

Rule.—*First find the common fraction by Art.* 316 ; *then reduce this fraction to a decimal.*

For problems, see Exercise XCV.

To reduce a Compound Number to a Decimal of a higher name (Second Method).

Example 1.—Let it be required to reduce 2 pks. 2 qts. 1 pt. to decimal of a bushel.

2)1 =1 pt.
8)2.5 =2 qts. 1 pt.
4)2.3125 =2 pks. 2 qts. 1 pt. =
.578125 bu.

Example 2.—Req. to reduce 7 oz. 3 dwts. 15 grs. to dec. of 1 lb.

24)15 =15 grs.
20)3.625 =3 dwts. 15 grs.
12)7.18125 =7 oz. 3 dwts. 15 grs. =
.5984375 lb.

Rule.—*Write the parts of the compound number in a column, that of the lowest name at the top. Divide this lowest by as many as will make one of the next higher, and annex the decimal thus formed to the whole number below. Divide this mixed number by as many of its kind as will make one of the next higher, and annex the quotient as a decimal to the whole number of the same name. Continue till the required name is reached.*

Note.—Beside those which follow, the problems under Art. 317 may be used for examples.

Exercise XCVI.

Reduce the following :

1–3. £17 10s. 9¾d., £2 5s. 7½d., £8 15s. 4¼d. to the decimal of £1.

4–6. 17 cwt. 3 qrs. 2 lb., 3 qrs. 9 lbs., 427½ lbs. to cwts.

7–9. 17 lbs. 6 oz., 2 qrs. 1 lb., 3 cwt. 1 qr. to dec. of a ton.

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10. 5 days 12 hrs. 25 mins. 37 secs. to dec. of a week.
11, 12. 7 oz. 4 dwt., 9 oz. $2\frac{1}{8}$ drams to decimal of a lb.
13. 8 gals. 2 qts. 1 pt. 3 gills to the dec. of a bbl.
14. A cubic foot (1728 in.) to the dec. of a bush. (2150.4.)
15. 2 pks. 1 qt. to dec. of a cubic foot.
16. 1 wine gal. to dec. of a dry gal. (268.8 in.)
17. 1 wine gal. to dec. of an imperial gal. (277.3 in.)

NOTE.—The last four problems should be solved by reducing the numbers to their common unit (cu. in.)

EXERCISE XCVII.

RULE.—*Reduce each quantity to the denomination whose price is given, and multiply the denominate number by the price. The price should be considered as an aliquot part of a dollar, or as made up of aliquot parts of a dollar.*

Required the cost of the following:

1. 15 lbs. 8 oz. 4 drs. of butter at $37\frac{1}{2}$c. a lb.
2. 18 lbs. 7 oz. of cheese at $12\frac{1}{2}$c. a lb.
3. 9 lbs. 2 oz. 8 dwts. 5 grs. of silver at \$13.87$\frac{1}{2}$c. a lb. Tr.
4. 6 cwt. 28 lbs. sugar at \$8.37$\frac{1}{2}$c. a cwt.
5. 16 gals. 2 qts. of sirup at \$1.25 a gal.
6. 19 gals. 2 qts. of oil at $62\frac{1}{2}$c. a gal.
7. 1 oz. 3 drs. of cloves at $16\frac{2}{3}$c. an oz.
8. 7 oz. 5 drs. of cinnamon @ 25c. an oz.
9. 12 t. 2 cwt. 3 qrs. of hay at \$14.87$\frac{1}{2}$ a ton.
10. $17\frac{1}{2}$ bu. flaxseed at \$2.16$\frac{2}{3}$ a bu.
11. $29\frac{2}{3}$ bu. oats at $66\frac{2}{3}$c. a bu.
12. 75 gals. 1 qt. 1 pt. of wine at \$4.75 a gal.
13. 19 yds. 3 qrs. of cloth at \$3.12$\frac{1}{2}$ a yard.
14. 224 yds. 1 qr. silk at \$2.06$\frac{1}{4}$ a yard.
15. 536 bu. 2 pks. rye @ $62\frac{1}{2}$c. a bushel.
16. 319 bu. 1 pk. cloverseed at \$10.12$\frac{1}{2}$ a bushel.
17. 114 bu. 3 pks. wheat at \$1.37$\frac{1}{2}$ a bushel.
18. 443 bu. 2 pks. beans at \$1.12$\frac{1}{2}$ a bushel.

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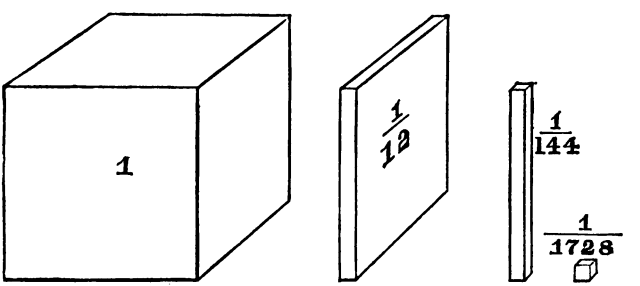
19. 19 dozen and 9 eggs at 18¾c. a doz.
20. 18 dozen and 4 eggs at 16⅔c. a doz.
21. 17 lbs. 8 oz. of feathers at 66⅔c. a lb.
22. 14 rods 2 yds. 1 ft. of wall, laid at $3.12½ a rod.

MEASUREMENT OF SURFACES AND SOLIDS.



319. If a unit be divided into 12ths, the parts may be considered as of the *first* division. These parts are called *primes*. They are marked thus, 12′. If one of these 12ths or primes be divided into 12ths, the parts are called *seconds*, and are marked thus, 12″. A third division would lead to *thirds*, marked thus, 12‴. These numbers are fractions and are called duodecimals. Their denominators are not written, but are known by the indices written at the right. Thus:

$$6'' = \frac{6}{12 \times 12}, \text{ or } \frac{6}{144}.$$

There are as many times 12 (12 taken as a factor) in the

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denominator of a duodecimal as it has indices. These fractions were formerly in common use and were applied by a rule, known as CROSS MULTIPLICATION, to the measurement of surfaces and solids.

The addition and subtraction of duodecimals may be performed by the rules for the addition of D. N., the scale being 12. Thus, 12 thirds make one second ; 12 seconds make one prime ; and 12 primes make a unit.

The multiplication of duodecimals is thus illustrated :

Let it be required to multiply $a2'\ b3''$ by $c4'\ d5''$.

	$2'$	$3''$
	$4'$	$5''$
	$10'''$	$15''$
$8''$	$12'''$	
$9''$	$11'''$	$3''''$

$$a\frac{2}{12}+b\frac{3}{12\times12}$$

$$c\frac{4}{12}+d\frac{5}{12\times12}$$

For convenience of reference we have lettered the fractions a, b, c, and d.

$$d\times b=\frac{3}{12\times12}\times\frac{5}{12\times12}=\frac{15}{12\times12\times12\times12}$$

$$d\times a=\frac{2}{12}\times\frac{5}{12\times12}=\frac{10}{12\times12\times12}$$

$$c\times b=\frac{3}{12\times12}\times\frac{4}{12}=\frac{12}{12\times12\times12}$$

$$c\times a=\frac{2}{12}\times\frac{4}{12}=\frac{8}{12\times12}$$

This method is now obsolete. The work above, for instance, might represent the finding the area of a figure, one of whose sides is $2\frac{3}{12}$ inches, and the other $4\frac{5}{12}$ inches—the result appearing as $\frac{9}{144}+\frac{11}{1728}+\frac{3}{20736}$ of a square foot.

Modern Method.

$2\frac{3}{12}$ or $2\frac{1}{4}=\frac{9}{4}$; $4\frac{5}{12}=\frac{53}{12}$. $\frac{9}{4}\times\frac{53}{12}=\frac{159}{16}=9\frac{15}{16}$ sq. in.

This method by fractions is preferable.

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EXERCISE XCVIII.

1. Req. to find the volume of a load of wood 4 ft. 6 in. high, 3 ft. 8 in. wide, and 9 ft. 9 in. long.

NOTE.—Consider the inches as fractions of a foot.

Operation: $4\frac{1}{2} \times 3\frac{2}{3} \times 9\frac{3}{4}$

$\frac{3}{2} \times \frac{11}{3} \times \frac{39}{4}$

$\frac{1287}{8} = 160.875$ cu. ft. $= 1.26$ cords.

2. A rectangular block of marble is 9 ft. 2 in. long, 6 ft. 3 in. wide, and 2 ft. 4 in. thick. Required its contents. *Statement.* $9\frac{1}{6} \times 6\frac{1}{4} \times 2\frac{1}{3}$.

3. How many square feet in the floor of an apartment which is 21 ft. 6 in. long and 16 ft. 9 in. wide? *Statement.* $21\frac{1}{2} \times 16\frac{3}{4}$.

4. Required the product of 4.2′ by 3.5′.

5. Required the product of 4.3′ 2″ by 1.6′.

6. Req. prod. of 4 yds. 2 ft. 7 in. by 2 yds. 1 ft. 4 in.

Ex. 4 yds. 2 ft. 7 in. $=14$ ft. 7 in., or 175 in.
2 yds. 1 ft. 4 in. $=7$ ft. 4 in., or 88 in.
$175 \times 88 = 15400$ sq. in. $=106$ sq. ft. 136 in., or 11 sq. yds. 7 ft. 136 in.

7. A room is 18 ft. long, 17 feet 3 in. wide. Its walls are 13 ft. 4 in. high. If carpeting 3 ft. wide cost \$1.50, and paper 3 ft. 4 in. wide cost 30 cts.; what will it cost to carpet the floor and paper the walls? (See the following rule.)

Solution: $\$\frac{3}{2} \times \frac{18 \times 17\frac{1}{4}}{9} = \$1 \times 3 \times 17\frac{1}{4} = \$51.75.$

$30 \text{ cts.} \times (18 + 17\frac{1}{4}) \times 2 \times 13\frac{1}{3} \div (3\frac{1}{3} \times 3) = \$28.20.$

Find the areas of the floors described as follows:

8, 9. 20 ft. 2 in. by 16 ft. 4 in.; 25.6 in. by 15 ft. 6 in.

10, 11. 28 ft. 10 in. by 17 ft. 6 in.; 12 ft. 8 in. by 7 ft. 8 in.

12, 13. $14\frac{1}{2}$ ft. by $7\frac{1}{2}$ ft.; 17 yds. 1 ft. by 4 yds. 2 ft.

14–16. Req. the cost of carpeting a room at 75 cts. a square yard, if it is 14 ft. 8 in. by 10 ft. 3 in.; of a room 23 ft. 3 in. by 16 ft. 3 in.; of a room 31 ft. 4 in. by 21 ft. 3 in.

17. If a room is 36 ft. long and 12 feet broad, how much

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less will it cost to cover it with carpeting $\frac{3}{4}$ yd. wide at 75 cts. than with carpeting $\frac{7}{8}$ yd. wide @ $1.25?

To find the Area of the Four Walls of a Room.

Rule.—*Add the length and breadth, multiply their sum by 2, and multiply this product by the height.*

1. Req. the number of sq. yds. of paper required for the walls of a room which is $22\frac{1}{2}$ ft. long, 18 ft. wide, and 15 ft. high. $(\overline{22\frac{1}{2}+18}\times 2\times 15)$.

2. What is the surface of the four walls of a room $22\frac{1}{2}$ feet long, $16\frac{1}{2}$ feet wide, and 18 feet high?

3. Of a room $25\frac{1}{4}$ by $16\frac{1}{2}$ and $17\frac{1}{2}$ ft. high?

4. Of a room 15 ft. 6 in. by 13 ft. 4 in. and 12 ft. 9 in. high?

5. Of a room 22 ft. long, 15 ft. 4 in. wide, and 7 ft. 6 in. high?

6–10. A roll of paper is commonly 8 yards long and 18 inches wide; what will the paper for each of the five rooms last described cost at 40 cts. a roll?

11–15. Required the cost of carpeting each with Brussels at $2 a yard of 27 inches in width.

16, 17. If a room is 17.6 ft. long, 19.27 ft. wide, and 12.4 ft. high, what will pay for its carpet at $1.75 per sq. yd., and for papering it at 22 cts. a square yard?

18. If a girl covered the walls of her room with postage stamps, how many were needed, the apartment being $12\frac{1}{2}$ ft. long, $9\frac{1}{2}$ ft. wide, and 10 feet high, and having 2 doors each 6 ft. by 3 ft., and 4 windows each $5\frac{1}{2}$ ft. by 4 ft.? A stamp is 1 in. long and $\frac{3}{4}$ in. wide.

19. How many chairs will seat a room 57 ft. by 32 ft., if at one end there be a clear space of 12 ft., if an aisle of 4 ft. be made in the middle, if each chair is $1\frac{1}{2}$ ft. wide, and if the chairs are so ranged that each rank take 2 ft., viz., 2 ft. from back to back? *Ans.* 420.

20. Find the product of these three factors, viz., 2 yds. 1 ft., 4 yds. 9 in., 2 ft. 8 in. $(7\times\frac{8}{3}\times\frac{51}{4}$ ft.)

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PERCENTAGE IN A NUTSHELL.

a. ***Percentage relates to estimates made in hundredths.***

b. **Per cent.** signifies by the hundred: as commonly employed, it means *hundredths*. Its sign is %. Thus, 2 per cent. = 2%.

The following are equivalents:

$\frac{1}{100}$,	.01,	1 per cent.,	1 %.
$\frac{2}{100}$,	.02,	2 "	2 %.
$\frac{2\frac{1}{2}}{100}$,	.025,	$2\frac{1}{2}$ "	$2\frac{1}{2}$%.
$\frac{1}{200}$,	.005,	$\frac{1}{2}$ "	$\frac{1}{2}$ %.

c. In business it is customary to compare the gain or loss with the *cost*, the interest with the *principal*, the tax with the *valuation*, the premium with the *sum insured*, the waste with the *original quantity*, etc., etc.

NOTE.—At this point review carefully Art. 311.

d. As any number may be considered as a fraction of any other number, and as all common fractions may be reduced to hundredths, it follows that **any number may be regarded as a certain per cent. of another number.**

e. The number of which the per cent. is taken is called **the base.** It is a *multiplicand.* The **per cent.**, generally called **the rate,** indicates the number of hundredths to be taken. It is a *multiplier* and is, of course, an abstract number. The *product* of the base and the per cent. is called the **percentage.**

Base × per cent. = **Percentage.**
Base = Percentage ÷ per cent.
Per cent. = Percentage ÷ Base.

The words italicized in paragraph *c* signify base.

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Per cent. is always expressed *decimally;* hence pupils should never neglect to **point off results.**

As 100 hundredths of a number equals the given number, so 110%, or 150%, of the base, is more than the base. On the other hand, when the percentage is $\frac{1}{2}\%$ of the base, it is but $\frac{1}{200}$ of it. Pupils must exercise some **common sense** in determining the result. One who relies wholly on rules may get confused, and, having made errors, will fail to perceive them. Answers should seem reasonable.

1. 2% of 1000 = ? *Analysis:* $\frac{2}{100} \times 1000 = 2 \times 10 = 20$.

2. 4% of \$500 = ? *Analysis:* $\frac{1}{100}$ of \$500 = \$5. And $\frac{4}{100}$ of \$500 = \$20.

3. $\frac{1}{10}\%$ of 400 = ? 1% of 400 = 4, and $\frac{1}{10}\%$ of 400 = .4.

4. $1\frac{1}{10}\%$ of 48 = ? 1% of 48 = .48, and $\frac{1}{10}\%$ of 48 = .048; these percentages added = .48 + .048 = .488.

5. 200% of \$6 = ? 100% of \$6 = \$6, and 200% of the same = \$12.

6. $1\frac{1}{9}\%$ of \$45 = ? $\frac{10}{9}\%$ will equal $\frac{10}{9} \times 1\%$ of anything. As 1% of \$45 = \$.45, $\frac{1}{9}$ of \$.45 = \$.05, and $\frac{10}{9}$ will equal \$.05 × 10 = \$.50, or 50 cents.

The statement in brief is $\frac{\frac{10}{9}}{100}$ of \$45 = $\frac{10}{900} \times$ \$45 = \$.50.

In the following questions, let the first ten letters stand for *rates*, and those following *j*, for *bases*. The respective products will equal **percentages** to be represented thus: a × K = \$9.60; a × M = .24 doz.; h × P = 1221 gals., etc.

a = 2%, b = 12%, c = $\frac{1}{2}\%$, d = $3\frac{1}{3}\%$, e = 110%, f = $33\frac{1}{3}\%$, g = 25%, h = $137\frac{1}{2}\%$, i = 1000%, j = $\frac{1}{4}\%$.

K = \$480, L = £250, M = 12 doz., N = 120 francs, O = 2000 metres, P = 888 gals.; Q = \$12.50, R = \$3.10, S = 2s. 6d.; T = 4 yrs. **100 problems.** Multiply every rate by every base.

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WHEN THE COST AND SELLING PRICE ARE GIVEN, TO FIND THE GAIN OR LOSS.

Find the difference between the selling price and the cost.

1. A coat cost \$12, and was sold for \$6; l = ?
2. A watch " \$40, " " \$15; l = ?
3. A knife " 50 cts. " " 75 cts; g = ?
4. A horse " \$120, " " \$75; l = ?
5. A house " \$12000, " " \$8000; l = ?
6. A book " \$1, " " 10 cts.; l = ?
7. A lamp " \$1.75 " " \$2.24; g = ?
8. A dog " \$2, " " \$10; g = ?
9. A cart " \$14, " " \$20; g = ?
10. A goat " \$10, " " \$1; l = ?

TO FIND THE GAIN OR LOSS PER CENT (RATE).

If necessary, find the gain or loss by subtraction as above. Divide the gain or loss by the cost. The quotient will express the rate in hundredths.

Thus, in problem 1, above, $12-6=6$; $\frac{6}{12}=.50$ or 50%.

Take the last 10 problems for an exercise.

1st *ans.*, 50%; 2d *ans.*, $62\frac{1}{2}$%; 3d *ans.*, 50%; etc., etc.

An insurance policy guarantees a certain sum in case of loss for a percentage called the premium.

Salesmen and Agents are paid percentages, called commissions, on the values of goods changing hands; Stock Brokers, on the par value of stocks or bonds bought or sold (see p. 183).

THE % BEING GIVEN, TO FIND THE PERCENTAGE, *i.e.*, GAIN OR LOSS, INTEREST, COMMISSION, TAX, PREMIUM, ETC.

Multiply the base (viz., the cost, principal, sum of money, valuation, or sum insured) by the rate. The selling price = cost + gain, or cost — loss.

1–30. A tradesman determined to sell his dry goods at a profit of $16\frac{2}{3}$%, his hardware at a gain of 25%, and his groceries at an advance of $12\frac{1}{2}$%. The cost of each of the various articles was as follows (See columns *a*, *b*, and *c*):

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	a.	*b.*	*c.*
1.	Sugar, per lb., 10c.	Linen, per yd., 42c.	Knives @$0.60
2.	Coffee, " 24	Muslin, " 24	Scissors @ .32
3.	Raisins, " 16	Calico, " 12	Pins @ .08
4.	Rice, " 12	Silk, " 1.32	Thimbles @ .44
5.	Starch, " 8	Cassi'ere, " 1.80	Nails @ .08
6.	Tea, " 72	Alpaca, " 60	Axes @ .80
7.	Cloves, per oz. 12	Br. cloth, " 3.00	Hoes @ .72
8.	Nutmegs, " 10	Canvas, " 18	Spades @ 1.00
9.	Flour, bbl., 8.00	Buttons, doz., 24	Rakes @ .56
10.	Molas's, gal. 56	Socks, pr. 20	Scythes @ .96

What are the respective selling prices? (30 problems.)

NOTE.—Mark the columns *a*, *b*, and *c*. Thus, the selling price of spades will be opposite 8 *c*, and will equal 1.25 or 8 *c* = $1.25.

After solving the problems as decimals, work them as questions in common fractions. $16\frac{2}{3}\% = \frac{1}{6}$, etc. This last mode is much more expeditious. Thus, the answers to 1 *a*, 2 *a*, 3 *a*, 4 *a*, 5 *a*, and 6 *a* are $10 + 1\frac{1}{2}$; $24 + 3$; $16 + 2$; $12 + 1\frac{1}{2}$; $8 + 1$; $72 + 9$.

1–42. A man loaned each of the following sums at every one of the following rates of interest; what were the respective annual payments of **interest**? (Interest = Pr. × r.)

1. $450, 2. $4500, 3. $4.50, 4. $1625, 5. $16.25, 6. $175.25, 7. $45.725, at *a*. 4%, *b*. 6%, *c*. 7%, *d*. 9%, *e*. $3\frac{1}{2}\%$, *f*. 12%.

Register the answers thus: 1*a*, $18. 2*a*, $180. 3*a*, .18. 1*b*, $27. 2*b*, $270.

1–10. An insurance company established rates as follows: For brick houses safely located, $\frac{1}{4}\%$; for well-built frame houses in blocks, $\frac{1}{2}\%$; for frame barns, shops, and stables, $1\frac{1}{2}\%$. What was the **premium** on each of the following policies?

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1.	1 brick house,	$4500	6.	1 shop,	$1800
2.	1 " "	8200	7.	1 barn,	2700
3.	1 " "	3900	8.	1 stable,	2800
4.	1 frame "	2700	9.	1 shop,	2900
5.	1 " "	2900	10.	1 brick house,	2700

1–5. **Commission.**—What were the sums paid to a salesman, his commission being $2\frac{1}{2}\%$, and his sales in successive years being $96,000, $108,000, $223,500, $146,900, $182,400?

1. $96000 × .025 = $2400, etc.

1–5. The **daily register and attendance** of a certain class for one week was as follows: 40—36, 42—36, 41—35, 38—34, 39—37. What decimals will express the daily per cents. of attendance?

$\frac{36}{40}$, $\frac{36}{42}$, $\frac{35}{41}$, $\frac{34}{38}$, $\frac{37}{39}$ = .90, 85.7+, etc.

What was the per cent. for the week? *Ans.* 89%.

Tax Estimates.

1–5. Certain houses were assessed $3\frac{1}{2}\%$ on these valuations, viz.: $4500, $14600, $12350, $19420, $3260. Req. the **taxes.**

To find the Cost, the Gain or Loss % being given.

Divide the selling price by 1 *plus or minus the gain or loss %.*

Anything may be considered as consisting of 2 halves, 10 tenths, or 100 hundredths. The cost = 100 hundredths of itself. The selling price = the cost plus the gain, or the cost minus the loss. If the gain % is 10, the selling price will = 110% of the cost. Then $\frac{\text{S. Price}}{110} \times 100 = \text{cost}$, or $\frac{\text{S. Price}}{1.10} = \text{cost}$.

The property on a model farm was sold at the rates specified; what was the first cost of each animal or article?

Horses at $240, g. 20%; cows @ $120, g. 40%; oxen at $110, l. 5%; sheep @ $4.52, g. 13%; ploughs @ $8.20, g. 4%; mowing-machines @ $70, l. 30%; churns @ $12, l. 20%.

1–10. What **principal** will yield $725 annual interest, the

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rates being as follows: 4%; 5%; 6%; 7%; 8½%; 12½%; 5½%; 3½%; 2¼%; 6⅛%? Problem 1 appears as follows:

1. \$725 is $\frac{1}{100}$ of what number? $\frac{\$1811}{4} \times 100 = \$18125$.

To COMPUTE INTEREST.

What is the interest on \$624 for 2 y. 3 m. 12 d. @ 6%?

Solution.—$2 + \frac{3}{12} + \frac{12}{360} = 2\frac{17}{60}$, or

$624 \times \frac{6}{100} \times 2\frac{17}{60} = 37.44 \times 2 + (.624 \times 17) = \$74.88 + 10.61 = \$85.49$.

RULE.—*Reduce months and days to a fraction of a year. Multiply the interest for 1 year by the number of years. Months and days may be considered as aliquot parts of a year.*

1–125. It is required to compute the interest on each of the following sums marked *a*, *b*, *c*, *d*, and *e*, at each of the rates lettered *m*, *n*, *o*, *p*, and *q*, for each of the different periods *r*, *s*, *t*, *u*, and *v*. Indicate each result by means of the respective letters.

a.	\$120	*m*.	4%	*r*.	2 yrs. 1 m. 12 d.
b.	\$435.50	*n*.	5%	*s*.	3 " 4 " 18 "
c.	\$1269	*o*.	6%	*t*.	5 " 5 " 0 "
d.	\$18.24	*p*.	7%	*u*.	4 " 7 " 0 "
e.	\$1824	*q*.	8½%	*v*.	1 " 6 " 6 "

Mark the 1st result, *amr*; the 2d, *ams*; 3d, *amt*, etc.
" 6th " *anr*; the 7th, *ans*; 8th, *ant*, etc.
" 26th " *bmr*; the 27th, *bms*; 31st, *bnr*, etc.
" 51st " *cmr*; the 56th, *cnr*; 61st, *cor*, etc.
" 101st " *emr*; the 106th, *enr*; 125th, *eqv*.

The Amount = the Principal + the Interest.

Discount.—The present worth of a sum due at a future time equals the quotient of the sum divided by the amount of \$1. The discount equals the debt less the present worth. The P. W. of \$400 due 6 months hence = \$400 ÷ \$1.035 = \$386.47. The D. = \$13.53. P. W. *amr*? *Ans.* \$110.63.

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KEY.

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ANSWERS TO PROBLEMS

IN THE

STANDARD ARITHMETIC.

Number One.

It has not been thought necessary to furnish the answers to the introductory questions. A few problems in the subsequent exercises have been left unanswered. These will serve as test questions. It is the opinion of the author that the habit of self-reliance resulting from the solution and proving of unanswered questions is worth more than the convenience which answers afford.

Exercise XVI.—(1) 170 mi. (2) $443. (3) 70c. (4) 4507. (5) 20704. (6) 6585. (7) 978 a. (8) 168 yrs. (9) 129 yrs. (10) 18 yrs. (11) 23c. (13) 8. (14) 16. (15) 24. (16) 19 ft. (17) 380 yrs. (19) Lost 2c. (20) 25 mi., 50 mi., 75 mi., 100 mi. (23) $20467. (26) John has 360, Jos. 120, Jerry 460.

XX.—(1) 9819114. (2) 15582507. (3) 12380622. (4) 37355325. (5) 2726784. (6) 1959552. (10) 63360 in. (11) 1577600640 in. (12) 22176000 ft. (13) 21120000 ft. (14) 19800000 ft. (15) 17952000 ft. (16) 12672000 ft. (17) 7920000 ft. (18) 1848000 ft. (19) 739200 ft. (21*a*) 20008 sq. ft. (21*b*) 1344 sq. ft. (23) 1913000 mills. (25) 128000 oz. (26*a*) 6000 d. (26*b*) 10560 d. (27*a*) 384 gi. (27*b*) 1568 gi. (27*c*) 6400 gi.

XXI.—(1) 634752. (2) 102. (3) 6720. (4) $15.65. (5) $112. (7) He got the lion's share. (8) None, for he still owed $529.60. (9) 6486480 chestnuts.

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XXII. AND *XXIII.*—The answers, if referred to by the pupil, would neutralize the whole advantage of the operation. Those in *Exercise XXIV.* are given to assist such pupils as may have no instructor.

XXIV.—(2) 387. (3) 658. (4) $208\frac{2}{21}$. (5) 7500. (6) 234. (7) 52.

XXV.—*Remainders* (1) 27. (2) 15. (3) 102. (4) 3. (5) 0. (6) 18. (7) 37. (8) 8. (9) 12. (10) 25. (11) 5. (12) 24. (13) 78. (14) 89. (15) 44. (16) 15. (17) 8. (18) 16. (19) 85. (20) 5.

XXVI.—*Remainders* (1) 243. (2) 24. (3) 2. (4) 0. (5) 300. (6) 1680. (7) 50. (8) 1400. (9) 2000. (10) 110. (11) 2000. (12) 950. (13) 27200. (14) 11000. (15) 2680.

XXVII.—(1) 4. (2) 4. (3) 12. (4) $19\frac{1}{5}$. (5) 5.

XXVIII.—(1) $6641\frac{118}{365}$. (2) $69596\frac{888}{1170}$. (3) $91\frac{19}{65}$. (4) $497\frac{793}{1400}$. (5) $3677010\frac{108}{125}$. (6) $1479710\frac{19}{125}$. (7) $4167\frac{35}{189}$. (8) $363\frac{961}{1176}$. (9) 333. (10) 16. See Art. 138 and PROVE (10), (11), etc.

XXIX.—(1) 815. (2) $725\frac{23}{45}$ in. (3) $524. (4) 29 yrs., 71 yrs. (5) 648501. (6) 388601. (7) $3561746\frac{1}{84}$. (8) 82247084. (9) $22673819\frac{105}{172}$. (10) 231201. (11) 23925. (12) 53. (13) $157\frac{1}{8}$. (14) 16737. (15) 1083. (16) 0. (17) A 18, B 57, C 33. (18) $300\frac{15}{17}$ ds. (19) 2000 Ger., 3000 Ts., 7000 speak both. (20) 524. (21) 567342. (22) 1085906. (23) $10. (24) 6s. 4d. (25) 1890 rs. (26) 215 m., A 301, B 289. (27) 5 yrs. (28) 12 yrs. (29) $655. (30) 220 A. (31) $121. (32) $130\frac{12}{17}$. (33) $994. (34) $87. (35) 276. (36) 162 c. (37) $256\frac{1}{4}$ lbs. (38) 96 m., 984 m., (39) 15 [work back from 2 by opposite process]. (40) 251. (41) 0. (42) $54. (43) $10\frac{65}{196}$ bbls. (44) $81.

XXX.—(2) 630. (3) 1440. (4) 1260. (5) 1680. (6) 3465. (7) 64. (8) 180. (9) 1440. (10) 1575. (11) 180. (12) 480. (13) 1406000. (14) 8736. (15) 1377. (16) 1680. (17) 3570. (18) 15640. (19) 44850. (20) 252.

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(21) 6930. (22) 416. (23) 318482199. (24) 840 qts. (25) 120 [reduce all to qts.] (26) $21. (27) 7007. *Prove* remaining examples.

XXXII.—(1) $3\frac{1}{10}$. (2) $9\frac{1}{3}$. (3) $7\frac{3}{8}$. (4) $2\frac{5}{8}$. (5) $7\frac{4}{5}$. (6) $4\frac{7}{9}$. (7) $7\frac{3}{7}$. (8) $9\frac{3}{5}$. (9) $9\frac{3}{8}$. (10) $180\frac{3}{4}$.

XXXIII.—(1) $\frac{157}{10}$. (2) $\frac{151}{12}$. (3) $\frac{38}{17}$. (4) $\frac{377}{125}$. (5) $\frac{1007}{10}$. (6) $\frac{48}{5}$. (7) $\frac{54}{13}$. (8) $\frac{353}{13}$. (9) $\frac{2605}{59}$. (10) $\frac{50001}{50}$. (11) $\frac{25}{4}$. (23) 5 pts. (24) 9 pts. (25) 12 pts. (26) 25 q. (27) 33 q. (28) 67 q. (29) 79 q. (30) 51 d. (31) 61 d. (32) 79 d. (33) 116 d. (34) 43 di. (35) 77 di. (36) 96 di. (37) 455 di. (38) 19 q. (39) 22 q. (40) 29 q. (41) 85 gi. (42) 122 gi. (43) 63 gi. (44) 70 gi. (45) 65 pks. (46) 82 pks. (47) 77 pks. (48) 99 pks. (49) 28 fur. (50) 97 fur. (51) 129 fur. (52) 109 fur.

XXXIV.—(1) $\frac{8}{11}$. (2) $\frac{3}{4}$. (3) $\frac{134}{143}$. (4) $\frac{3}{5}$. (5) $\frac{3}{7}$. (6) $\frac{6}{13}$. (7) $\frac{3}{5}$. (8) $\frac{9}{13}$. (9) $\frac{7}{5}$. (10) $\frac{81}{135}$. (11) $\frac{7}{11}$. (12) $\frac{5}{19}$. (13) $\frac{1}{6}$. (14) $\frac{14}{55}$. (15) $\frac{3}{4}$. (16) $\frac{3}{5}$. (17) $\frac{3}{5}$. (18) $\frac{2}{3}$. (19) $\frac{2}{5}$. (20) $\frac{2}{5}$. Prove those remaining.

XXXV.—(1) $\frac{576}{12}$. (2) $\frac{156}{12}$. (3) $\frac{1152}{12}$. (4) $\frac{12012}{12}$. (5) $\frac{948}{12}$. (6) $\frac{114}{19}$. (7) $\frac{171}{19}$. (8) $\frac{228}{19}$. (9) $\frac{285}{19}$. (10) $\frac{2823}{19}$.

XXXVI.—(1) $\frac{6}{8}$, $\frac{15}{20}$, $\frac{21}{28}$, or $\frac{36}{48}$. (2) $\frac{21}{27}$, $\frac{28}{36}$, $\frac{35}{45}$, or $\frac{48}{63}$. (3) $\frac{22}{55}$, $\frac{26}{65}$, $\frac{40}{100}$, or $\frac{52}{130}$. (4) $\frac{10}{16}$, $\frac{15}{24}$, $\frac{25}{40}$, or $\frac{35}{56}$. (16) $\frac{16}{48}$, $\frac{8}{12}$, $\frac{4}{6}$, or $\frac{2}{3}$. (17) $\frac{27}{81}$, $\frac{9}{27}$, $\frac{3}{9}$, or $\frac{1}{3}$.

XXXVII.—(1) $\frac{6}{8}$. (2) $\frac{4}{9}$. (3) $\frac{7}{36}$. (4) $\frac{10}{21}$. (5) $\frac{15}{56}$. (6) $\frac{2}{65}$. (7) $\frac{27}{54}$. (8) $\frac{3}{10}$. (9) $\frac{74}{1}$. (10) $\frac{6438}{49}$. (11) $\frac{725}{56}$. (12) $\frac{107}{900}$. (13) $\frac{1}{4}$. (14) $\frac{1}{504}$. (15) $\frac{1}{63360}$. (16) $\frac{1}{5760}$. (17) $\frac{1}{32}$. (18) $\frac{1}{10000}$.

XXXIX.—(1) 8. (2) $26\frac{4}{5}$. (3) 9. (4) $\frac{8}{15}$. (5) $13\frac{1}{9}$. (6) $17\frac{6}{10}$. (7) $14\frac{1}{8}$. (8) $14\frac{1}{2}$.

XL.—(1) $1\frac{53}{60}$. (2) $1\frac{209}{252}$. (3) $5\frac{143}{252}$. (4) $6\frac{887}{2340}$. (5) $7\frac{53}{88}$. (6) $3\frac{19}{64}$. (7) $4\frac{405}{3136}$. (8) $2\frac{7}{300}$. (9) $11\frac{41}{60}$. (10) $10\frac{102}{266}$.

XLI.—(1) $\frac{20}{91}$. (2) $\frac{19}{90}$. (3) $\frac{16}{48}$. (4) $\frac{10}{21}$. (5) $\frac{15}{44}$. (6) $\frac{14}{45}$. (7) $\frac{17}{30}$. (8) $\frac{19}{48}$. (9) $\frac{30}{200}$. (10) $\frac{105}{500}$.

XLII.—(1) $\frac{24}{35}$. (2) $1\frac{1}{8}$. (3) $1\frac{11}{35}$. (4) $1\frac{7}{48}$. (5) $1\frac{5}{9}$. (6) $5\frac{7}{22}$. (7) $\frac{57}{60}$. (8) $8\frac{11}{45}$. (9) $13\frac{13}{44}$. (10) $17\frac{11}{60}$.

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XLIII.—(11) $\frac{13}{30}$. (12) $\frac{35}{36}$. (13) $1\frac{5}{24}$. (14) $142\frac{5}{9}$. (15) $41\frac{14}{15}$. (16) $17\frac{20}{99}$. (17) $\frac{1}{15}$. (18) $\frac{1}{20}$. (19) $9\frac{11}{12}$. (20) $17\frac{5}{14}$. (21) $\frac{27}{280}$. (22) $\frac{188}{231}$.

XLIV.—(1) $\frac{1}{30}$. (2) $\frac{1}{110}$. (3) $\frac{4}{221}$. (4) $\frac{3}{28}$. (5) $\frac{2}{63}$. (6) $\frac{5}{24}$.

XLV.—(1) $\frac{6}{63}$. (2) $\frac{60}{50}$. (3) $\frac{65}{14}$. (4) $\frac{96}{84}$. (5) $\frac{49}{120}$. [Ans. capable of further reduction.]

XLVI.—(1) 3. (2) 64. (3) 10. (4) 6. (5) $\frac{1}{2}$. (6) 1. (7) $12\frac{104388}{247120}$. (8) $117\frac{3295}{7820}$. (9) $\frac{3652}{10071}$. (10) $21\frac{34431}{92440}$. (11) \$53412.70$\frac{5}{8}$. (12) $3\frac{73}{262}$. (13) \$1834$\frac{3}{8}$. (14) \$4187$\frac{3}{8}$.

XLVII.—(3) 886. (4) 80561$\frac{2}{13}$. (5) 2043. (6) 1092$\frac{3}{4}$. (7) \$71$\frac{3}{4}$. (8) \$287$\frac{5}{8}$. (9) \$126. (10) 32$\frac{5}{8}$. (11) [62$\frac{1}{2}$c.= \$$\frac{5}{8}$, hence] \$5$\frac{5}{8}$. (12) \$43$\frac{1}{2}$. (13) \$86$\frac{7}{8}$.

XLVIII.—(1) 326. (2) 9$\frac{5}{7}$. (3) 70$\frac{1}{2}$. (4) 57$\frac{1}{2}$. (5) 31$\frac{1}{4}$. (6) 5$\frac{3}{4}$. (7) 27$\frac{1}{2}$. (8) 121$\frac{3}{4}$. (9) $7\frac{23}{35}$. (10) 361$\frac{1}{4}$.

XLIX.—(35) \$12$\frac{21}{50}$. (36) \$2$\frac{14}{25}$. (37) \$2$\frac{13}{16}$. (38) 39$\frac{13}{88}$ m. (39) 302$\frac{18}{28}$c. (40) \$11.72$\frac{7}{8}$ (41) \$20.01. (42) $20\frac{9901}{12660}$. (43) \$116$\frac{6}{7}$. (44) \$29$\frac{278}{3240}$. (45) 10c.

L.—(1) $\frac{3}{7}$. (2) $\frac{3}{16}$. (3) $\frac{15}{64}$. (4) $\frac{2}{27}$. (5) $\frac{2}{117}$. (6) $\frac{1}{14}$. (7) $\frac{3}{25}$. (8) $\frac{1}{50}$. (9) $1420\frac{7}{15}$. (10) 10$\frac{3}{8}$. (11) $\frac{1}{12}$. (12) 25$\frac{13}{30}$. (13) 12$\frac{28}{36}$. (14) $\frac{3}{28}$. (15) $\frac{5}{96}$. (16) $\frac{1}{30}$. (17) $\frac{1}{16}$. (18) $\frac{7}{2710}$. (19) $\frac{9}{26}$. (20) $\frac{5}{7}$. (21) $\frac{21}{148}$.

LI.—(1) 14$\frac{2}{3}$. (2) 17$\frac{1}{2}$. (3) 11$\frac{2}{3}$. (4) 31$\frac{1}{2}$. (5) 50. (6) 2. (7) 84. (8) $81\frac{9}{10}$. (9) 32$\frac{1}{2}$. (10) 135.

LII.—(1) 1. (2) 2. (3) 1$\frac{1}{2}$. (4) 3$\frac{1}{2}$. (5) 2. (6) 2$\frac{2}{3}$. (7) $\frac{8}{9}$. (8) 1$\frac{2}{3}$. (9) 1$\frac{1}{8}$. (10) $\frac{9}{10}$. (11) $1\frac{1}{11}$. (12) $1\frac{1}{13}$. (25) 1$\frac{3}{4}$. (26) $1\frac{1}{14}$. (27) $1\frac{7}{48}$. (28) $1\frac{1}{14}$. (29) $\frac{44}{63}$. (30) $\frac{15}{16}$. (31) $\frac{1}{14}$. (32) $\frac{2}{45}$.

LIII.—(1) $63\frac{866}{1583}$. (2) £8$\frac{3001}{4000}$. (3) \$10266$\frac{2}{3}$. (4*a*) $\frac{6}{12}$. (4*b*) $\frac{4}{12}$. (5) $\frac{7}{37}$, $\frac{14}{111}$, $\frac{9}{37}$. (6) $\frac{3}{6}$. (7) $\frac{9}{16}$. (8) $\frac{386}{2888}$. (9) 704 hours (as he travels $\frac{1}{704}$ of the distance in 1 hour). (10) 46$\frac{2}{3}$ times. (11) 18$\frac{34}{35}$ yds. (12) \$50$\frac{69}{760}$. (13) \$6.18$\frac{1}{2}$. (14) 2441$\frac{37}{49}$. (15) \$$\frac{234}{437}$. (25) 11$\frac{1}{9}$° Cent., 8$\frac{8}{9}$° Réau. (26) 45° F., 25° C. (27) 36° Fahr., 16° Réau. (28) 3$\frac{1}{4}$ dimes.

LIV.—(1–9) Five hundredths. Twelve hundredths. One hundred and twenty-two thousandths. Twelve thou-

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sandths. One hundred and twenty-two ten-thousandths. Thirty-five thousandths. Three tenths. Nineteen ten-thousandths. (10–18) Fifty-five hundredths. Ninety-nine hundredths. Sixty-four thousandths. Two and decimal five-tenths. Two and decimal five hundredths. Four and decimal one hundredth. Four hundred and seventeen thousandths. One and decimal nine hundredths. One hundred and ninety thousandths.

LV.—(1–7) .3, .25, .242, .0009, .00009, .0091, .000106. (8–12) .06, .00006, .0000066, .000000054, .00000004. (13–17) 10.6, 20.04, 7.005, 600.06, 82.000082. (18–21) 90.0000000091, 201000.4, 6000.000041, .7. (22–40) .0051, .0000007, .041, .02, .00043, .000029, 41.0, 29.64, .031, 44.03, 2.1, 25.003, 4.01, 31.45, 2.47, .351, 2.0019, .00005, 21.4. (41) $4\frac{16}{100}$. (42) $91\frac{664}{1000}$. (43) $3\frac{19}{100}$. (44) $1\frac{6}{10}$. (5) $5\frac{16881}{100000}$.

LVI.—(1) 1004. (2) 51.68. (3) 329.1. (4) .047. (5) 8.235. (11) 4968. (12) .418. (13) 325. (14) 51.9. (15) 824. (21) 49.6. (22) 4.2. (23) 31. (24) 42.6. (25) 81.46. (31) .004. (32) .090. (33) .560. (34) .204. (35) 3.569. (46) .000351. (47) .000049. (48) .000005. (49) .008216.

LVII.—Eighty-one hundredths. Eighty-one thousandths. Ninety-six ten-thousandths. Ninety-six hundredths. Ninety-six hundred-thousandths. Forty-nine ten-thousandths. Twenty-five hundredths. Eight and decimal forty-two hundredths. Thirty-six units and nine tenths.

LVIII.—(1) .500. (2) .025000. (3) .0040. (4) .010500000. (5) .40. (6) .00300, .1110000, $.1\frac{2}{10}$, .104, .200000, 4.1500. (19) $.1\frac{64}{100}$. (20) $.356\frac{82}{100}$.

LIX.—(1) $\frac{125}{1000}$. (2) $\frac{34}{1000}$. (3) $\frac{56}{100}$. (4) $\frac{25}{100}$. (5) $\frac{25}{100000}$. (6) $1\frac{4}{100}$. (7) $2\frac{5}{10}$. (8) $1\frac{16}{100}$.

LX.—(1) .18. (2) .066+. (3) .125. (4) .375. (5) .75. (6) .02083+. (7) .4761904+. (8) .111+. (9) .285714+. (10) .2. For explanation of probs. 2, 6, 7, 8, and 9, see Articles 241 and 242.

LXI.—(1) 487.41. (2) 18.6375. (3) $28.25. (7) 481608.0865. (8) 102909.417. (9) 248.872. (10) 1316.872.

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LXII.—(1) 700.584. (2) .04816. (3) 1.8. (4) 12.987. (5) 1.584. (6) .1782. (7) 8.36. (8) $1.205. (9) $112.86. (19) $16.25. (20) $35.30.

LXIII.—(1) $3.4375. (2) $1.0625. (3) $576.268. (12) $59.22. (14) $428.40. (16) 97.544. (17) 25.19775. (18) .00434472. (19) .0336. (20) 2.72646. (35) $1.3125. (36) $334.125. (37) $\frac{5}{2}\times\frac{3}{2}\times 12=477$c., or $4.77. (38) $4.25. (39) $10. (40) $\frac{1}{4}\times\$.18\times 16=\$5.04$.

LXIV.—(39) $428.79. (40) $638. (41) 451.083. (42) $7.64. (43) $104.693. (44) $1134.56. (45) $398.20. (46) $213.45. (47) $84.55.

LXV.—(1) 841.5559199. (2) 288.635. (3) 57.978. (4) 571.044. (5) 238.594. Prove the remaining problems by "casting out the 9s."

Subtraction: (1) .040. (2) .784. (3) 14.039. (4) 102.196. (5) 26.180. (6) .044. (7) 710.091905. (8) 3.3444. (9) 2.344623. (10) 17.93166.

Multiplication: (1) .048060. (2) 6455.333960. (3) 2.672352. (4) .068096. (5) 142.785720.

Division: [In case of a repetend in quot., place a period over the repetend, and, if it consist of a group, then over the first and last group figures. See Example 1.] (1) 18.7816. (2) 188.43421+. (3) 2.9457+. (4) 55.55. (5) 5.236. [In finding the "9 overs," count 1 extra when the quotient is not complete—thus: 4.2)12.372(2.9457+, the quot. 9 over is 0, but must be considered as 1. See prob. 3, above.]

LXVI.—(4) $423. (5) $827.20. (6) $12.94. (7) $11.625. (8) $116.875. (9) $68.75. (10) $15.125. (11) $22.6875. (12) $100. (13) $50. (14) $10.50. (15) $4.50. (16) $5.95. (17) $7.4375. (18) $10. (19) $20. (20) $12.0975.

LXVII.—(1) 1230d., 2460 ($\frac{1}{2}$d.), 4920 far. (2) 126.5d., 1518 ($\frac{1}{2}$d.), 3036 far. (3) 200 books. (4) 506 apples. (5) 92 mel.

LXVIII.—(1) $1930.50. (2) $231.66. (3) $241.312.

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(4) 734000 cent. (5) 120 fr. (6) $23.166. (7) 958300 cent. (8) 1510 fr. (9) 196400 cent.

LXIX.—(1) $4\times24=96$ gr. (2) 168 gr. (3) 216 gr. (7) $3\times20\times24=1440$ gr. (8) 9120 gr. (12) $2\times12\times20=480$ dwt. (13) 2880 dwt. (17) 11568 gr. (18) 2020 gr.

LXX.—(1) 64 qrs. (2) 96 qrs. (4) 224 oz. (8) 7168 drs. (9) 13312 dr. (12) 11000 lbs. (13) 42400 lbs. (15) 544272 drs. (20) 68032 oz. (21) $8228\frac{4}{7}$ oz.

LXXI.—(1) $13\times12=156$ ʒ. (2) 84 ʒ. (3) 228 ʒ. (8) 340 ℈. (9) 1008 ℈. (14) 240 gr. (15) 960 gr. (18) 69120 gr. (26) 96 dr. (27) $3500\times7000\div5760$, or $4253\frac{17}{36}$ lbs. apoth. (28) $1800\times5760\div7000$, or $1481\frac{1}{7}$ lbs. avoir.

LXXII.—(1) 20 grams. (2) 120 grams. (3) 180 grams. (4) 1200. (5) 2000. (6) 20000. (7) 500 kilos. (8) 12000 kilos. (9) 7707 kilos. (10) 54144 dr. av. (11) 118122 lbs. (12*a*) 61736 grs. (12*b*) 46202 grs. (12*c*) 19446.84. (13) 34374.142. (14) 15432098 gr. [15433159 exactly]. (15) 15432098, if 453.6 grams were the exact equivalent of 1 lb. avoir. (16) 56448 dr.

LXXIII.—(1) 276.8 mi. (2) 4290.4 mi. (5) 7040 rds. (13) 9378 ft. (14) 151476 in. (15) 79998 in. (18) 47520 in. (19) 5280 in. (20) 4950 in.

LXXIV.—(2) 3.937 in. $\times12=47.244$ in. (3) 13300 dm. (4) 32810 ft. (5) 32.81 ft. (6) 1000. (7) $43746\frac{2}{3}$ in. (8) $35075\frac{1}{11}$ rods. (9) 2652.81494. (10) 6213.7 mi., for 1 myriametre=6.2137, and 1000 mm. is 10000000 m.

LXXV.—(1) 63360 in. (2) 4000 l. (3) 1600 l. (4) 2500 l. (5) 1875 l. (6) 6272.64 in. (7) 8870.4 in. (8) 64357.92 in. (9) 3360 ch. (10) 78080 ch. (11) 200 l. (12) 481536 ft. (13) 832 r. (14) 424 r. (15) 257431.68 in.

LXXVI.—(1) 64 a. (2) 648 a. (3) 1000 a. (4) $784\frac{1}{8}$ sq. m. (5) 169361.28 sq. in. (6) 72600 sq. yds. (7) $72\frac{31}{40}$.

LXXVII.—(1) 20000 sq. m. (2) 40000 sq. m. (3) 227120.4 sq. ft. (4) 2392 sq. yds. (5) 8611.2 sq. yds. (6) 10943.4. (7) 263120 sq. yds. (8) 110000 c. ar. (9)

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4694.455 ar. (10) 1821.125 ar. (11) 50100 c. ar. (12) 1000100. (13) 1220400. (14) 42.007 a. (16) 37954.56 sq. r. (18) 96.42×119.6×9 sq. ft. (19) 6200 sq. in.

LXXVIII.—(1) 6195200 sq. yds. (3) 35978400 sq. ft. (5) 7372800 sq. rds. (7) 117612 sq. in. (9) $6400. (10) $290.40. (11) $309760. (13) 25899.66 ar. (14) 759× 640÷2.471 ha.

LXXIX.—(5) 870912 cu. in. (6) 321408. (7) 61027. (9) 93312. (11) 997÷1728×231 oz. av. (12) 263.18 lbs. (13) 296.296 cu. yd. (14) 296.296÷1.308 c. m.

LXXX.—(1) 1000 m. s. (2) 100 d. s. (3) 353.166× 1728 cu. in. (4) 1000 li. (5) 100000 c. l. (6) 61027.1÷ 268.8. (7) 22.7 gal. (8) 22.7×46. (9) 163289.3 l. (10) 610220÷2150.4.

LXXXI.—(1) 881.05 l. (2) 1127.74 (3) 242.19 l. (4) 3451 l. (5) 838.6 bbls. (6) .08386×2468 bbls. (7) 1702.5 bu. (8) 2837.5 bu. (9) 2837.5×9.84 bu. (10) 28.375×76.

LXXXII.—(1) (46×38×36) ÷ 2150.4 × 120 = $35.11. (2) $6788.57. (3) $192. (4) $520.71. (5) 51609.6 cu. in. (6) 18×1728÷33.6=3×216÷.7=925$\frac{5}{7}$ pints. (7) 21×27 ×1728÷33.6=? (8) 7704. (9) (8×6×$\frac{5}{6}$)×1728÷2150.4 =32.138 bu.

LXXXIII.—(15) $1653.75. (16) $202.56. (17) $49.125. (18) 32400 pts.

LXXXIV.—(1) 4200′. (2) 5460′. (4) 360′. (5) 12463′. (6) 1296000. (8) 2 hrs.

LXXXV.—(9) 1389722 sec. (11) 27956929.7 sec. (12) 5739$\frac{9}{14}$ wks.

LXXXVI.—(4) 270 lbs. (6) 172 lbs. (12) 13584 ft. (14) 864. (15) 548. (16) 6720. (17) 8640. (18) 800. (19) 20$\frac{5}{12}$.

LXXXVII.—(1) 1 m. 38 r. 2 y. 2 ft. 7. in. (2) 2 a. 32 sq. rd. 29 sq. yd. 7 sq. ft. 19 sq. in. (3) 354 a. 8 sq. ch. 9 p. 19 sq. l. (4) 352 cir. 27° 25′ 19″. (5) 11 bbl. 15 gal. 3 qt.

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1 pt. 3 gi. (6) 4 t. 8 cwt. 1 qr. 3 lb. 10 oz. 3 dr. (7) 29 lb. 8 ℥ 3 ʒ 1 ℈ 19 gr.

LXXXVIII.—(1) £1 7d. (3) £1 15s. (5) 671.79 fr. (6) 114 lb. 4 oz. 13 dwt. (9) 6 lb. 3 ℥ 6 ʒ 3 ℈. (11) 8 yds. 1 ft. 7 in. (12) 20 m. 21 sec. (13) 767 a. 2 roods 16 p. (14) 16 sq. yd. 7 sq. ft. (15) 1 c. 119 cu. ft. 609 cu. in. (16) 61 gal. 1 qt. (23) 272 a. 1 r. 39 p.

LXXXIX.—(5) 79 d. (6) 231 d. (7) 67 y. 9 m. 22d. (8) 58 y. 11 m. 9 d.

XC.—(3) £86 4s. (4) £118 10s. 6d. (5) £527 19s. 6d. (6) 52 lb. 4 oz. 16 dwt. 18 gr. (7) 122 lb. 3 oz. 5 dwt. 18 gr. (14) 87 t. 9 cwt. 1 qr. 5 lb. (15) 122 t. 9 cwt. 2 lb. (19) 189 y. 2 ft. 10 in. (20) 350 y. 2 ft. (24) 18 bu. 3 p. 2 qt. (25) 43 bu. (26) 121° 48″. (27) 272° 16′ 48″. (28) 1 cir. 33° 17′ 36″. (34) 250 lbs. 7 ℥ 2 ʒ 1 ℈ 5 gr. (35) 322 lbs. 2 ℥ 4 ʒ 15 gr. (36) 110 a. 3 roods 4 p. (37) 174 a. 12 p.

XCI.—(1) 3 a. 2 sq. ch. 8 p. (2) 3 lbs. 9 oz. (3) 4 lbs. 12 oz. [av.]. (4) £7 15s. (5) 11 fr. 25 c. (6) 4 m. 2 fur. (7) 11 k. m. 7 h. m. 5 decm. (8) 9 ch. 3 r. 2 p. (9) 7 ha. 25 ares. (10) 11 fath. 1 ft. 6 in. (11) 19 cu. yds. 20 cu. ft. 432 cu. in. (12) 1° 45′. (13) 3 w. 1 d. 18 hrs. (14) 6 st. 2 q. 5 mg. (15) 127 lit. 2 dl. 5 cl. (16) 21 bu. 3 pks. (17) 22 gal. 3 qts. (18) 4 m. 2 dm. 5 cm. (19) 17 kg. 7 decg. 5 g. (20) 12.18 sh. (21) 2 a. 1 sq. ch. 10 p. 416$\frac{2}{3}$ sq. l. (22) 2 lbs. 6 oz. (23) 3 lbs. 2 oz. (24) £5 3s. 4d. (25) 7 fr. 50 c. (26) 2 m. 6 fur. 26 r. 3$\frac{2}{3}$ yds. (27) 7 km. 8 hm. 3 dm. 3$\frac{1}{3}$ m. or 78333$\frac{1}{3}$ m. (28) 6 ch. 50 links. (29) 4 ha. 83$\frac{1}{3}$ ares. (30) 7 fath. 3 ft. (31) 13 cu. yd. 4 cu. ft. 864 cu. in. (32) 1° 10′. (33) 2 w. 1 d. 4 hrs. (34) 3 s. 1 q. 6 mg. 6$\frac{2}{3}$ kg. (35) 84 lit. 8 dl. 3$\frac{1}{3}$ cl. (36) 14 bu. 2 pks. (37) 15 gal. 1$\frac{1}{3}$ pt. (38) 2 m. 8 dm. 3$\frac{1}{3}$ cm. (39) 11 kg. 8$\frac{1}{3}$ decg. (40) 8 quires 12 sh. The quotients arising from dividing by 11 must be found and the work proved by the pupil.

XCIV.—(1) $\frac{1}{42}$ d. (2) $\frac{1}{294}$ w. (3) $\frac{1}{15330}$ y. of 365 d. (4) $\frac{5}{27}$ yd. (5) $\frac{10}{297}$ r. (6) $\frac{1}{1188}$ ft. (7) 6$\frac{2}{3}$ in. (8) 1$\frac{5}{11}$ pt. (9) $\frac{8}{693}$ hhd. (10) $\frac{8}{11}$ qt. (13) 1$\frac{1}{10}$ p. (14) $\frac{81}{100}$ sq. ft.

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(15) $\frac{396}{625}$ in. (16) $\frac{1}{800}$ lb. (17) $\frac{1}{20}$ lb. Troy. (18) $\frac{1}{360}$ cir. (19) $\frac{1}{43200}$ cir. (20) $\frac{1}{1260}$ cir. (21) $\frac{1}{86160}$ cir. (22) $\frac{1}{864000}$ cir. (23) $\frac{1}{3780000}$.

XCV.—(1) .0106640625 cwt. (2) .0876736 lb. (3) .17986 lb. (4) .2627 a. (5) .8 m. (6) .875 bu.

XCVI.—(1) £17.540625. (2) £2.2816. (3) £8.7677+. (4) 17.77 cwt. (5) .84 cwt. (6) 4.275 cwt. (7) .008875 t. (8) .0255 t. (9) .1625 t. (10) .78825+. (11) 6 lb. (12) .77213 lb. apoth., or .5708 lb. av. (13) .27678+bbl. (14) .8036, nearly. (15) .661 cu. ft. (16) .8594 dry gal., nearly. (17) .833+imp. gal.

XCVII.—(1) $5.817. (2) $2.31. (3) $127.66. (4) $52.595. (5) $20.625. (6) $12.1875. (7) $.198, nearly. (8) $1.82+. (9) $180.531 [Solution, $12\frac{11}{30} \times 14\frac{7}{8}$]. (10) $37.916.

XCVIII.—(2) 133.68 cu. ft. (3) $360\frac{1}{8}$ sq. ft. (4) 14.237 (5) $6\frac{57}{144}$. (14) $12.53. (15) $31.48. (16) $110.97. (17) Dif. in price of a sq. yd. of carpeting is $\$\frac{3}{7}$, ∴ Ans. $20.57. (18) 41472 stamps.

PAGE 180.—Ka, $9.60. La, £5. Ma, 24 doz. Na, 2 fr. 40 cent. Oa, 40 m. Pa, 17.76 gals. Qa, 25 cts. Ra, $.062. Sa, 2.4 far. Ta, 29.22 days. Kb, $57.60. Lb, £30. Mb, 1.44 doz. Nb, 14 fr. 40 cent. Ob, 240 m. Pb, 106.56 gals. Qb, $1.50. Rb, $.372. Sb, 3d. 2.4 far. Tb, 175.2 days. Kc, $2.40. Lc, £2 5s. Mc, .06 doz. Nc, 60 cent. Oc, 10 m. Pc, 4.44 gals. Qc, $6\frac{1}{4}$ cts. Rc, $.0155. Sc, .6 far. Tc, 7.305 days. Kd, $16.80. Ld, £8 15s. Md, 5.04. Nd, 4 fr. 20 cent. Od, 70 m. Pd, 31.08 gals. Qd, $.4375. Rd, $.1085. Sd, 10.85d. Td, 51.135 days. Ke, $528. Le, £275. Me, 13.2 doz. Ne, 132 fr. Oe, 2200 m. Pe, 976.8 gals. Qe, $13.75. Re, $3.41. Se, 2s. 9d. Te, 4 yrs. 146 days. Kf, $\$480$, or $160. Lf, £83 6s. 8d. Mf, 4 doz. Nf, 40 fr. Of, $666\frac{2}{3}$ m. Pf, 296 gals. Qf, $\$4.16\frac{2}{3}$. Rf, $\$1.03\frac{1}{4}$. Sf, 10d. Tf, 1 yr. 4 m. Pupils to solve and prove the 40 remaining prob-

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lems of this exercise. Strict proof is either the opposite process, or an entirely different process.

Page 181.—*Gain or loss per cent:* (1) A coat which cost \$12 was sold for \$6; loss %, 50%. (2) 62½%. (3) 50%. (4) 37½%. (5) 33⅓%. (6) 90%. (7) 21⅞%. (8) 400%. (9) 42⅞%. (10) 90%.

Page 182.—Column *a:* (1) 11¼ c. (2) 27c. (3) 18c. (4) 13½c. (5) 9c. (6) 81c. (7) 13½c. (8) 11¼c. (9) \$9. (10) 63c.

Column *b:* (1) 49c. (2) 28c. (3) 14c. (4) \$1.54. (5) \$2.10. (6) 70c. (7) \$3.50. (8) 21c. (9) 28c. (10) 23⅓c. Pupils will solve and prove problems in col. *c*.

Annual Interest: 1*a*, \$18. 2*a*, \$180. 3*a*, 18c. 4*a*, \$65. 5*a*, 65c. 6*a*, \$7. 7*a*, \$18.29. 1*b*, \$27. 2*b*, \$270; etc. 1*e*, \$15.75. 2*e*, \$157.50. 3*e*, 15¾c. 4*e*, \$56.87½. 5*e*, 56¼c. 6*e*, \$6.125. 7*e*, \$16. The answers to 1*f*, 2*f*, etc., are not given.

Page 183.—*Insurance:* (1) \$4500@1%=\$45; at ⅛ of 1%, \$9. (2) \$16.40. (3) \$7.80. (4) \$13.50. (5) \$14.50. (6) \$27. (7) \$40.50. (8) \$42. (9) \$43.50. (10) \$5.40.

Commission: (2) \$2700. (3) \$5587.50. (4) \$3672.50. (5) \$4560.

Per cent. of attendance: (1) 90%. (2) 85.7%. (3) 85.3%. (4) 89.47%. (5) 94.87.

Taxes: (1) \$140. (2) \$454.22. (3) \$384.22. (4) \$604.18. (5) \$101.42.

Cost: (1) \$200. (2) \$85.71. (3) \$115.79. (4) \$4. (5) \$7.88. (6) \$100. (7) \$15 [thus, in the last, or 7th example, \$12 equals 100%—20%, or 80% of the cost—*i.e.*, \$12 is $\frac{80}{100}$ of $\$12 \times \frac{100}{80}$, or \$15.

Page 183-4.—*To find the Principal*—the interest, or percentage, being given, and also the rate, or %: (2) \$725 is 5% of \$725÷.05=\$14500. (3) \$12083.33. (4) \$10357.14. (5) \$8529.41. (6) \$5800. (7) \$13181.81. (8) \$20714.29. (9) \$32222.22. (10) \$11447.37.

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PAGE 184.—*Interest:* *amr*, $10.16. *ams*, $16.24. *amt*, $26. *amu*, $22. *amv*, $7.20. *anr*, $13.25. *ans*, $20.30. *ant*, $32.50. *anu*, $27.50. *anv*, $9. *aor*, $15.24. *aos*, $24.36. *aot*, $39. *aou*, $33. *aov*, $10.92. *bmr*, $36.87. *bms*, $58.94. *bmt*, $94.36. *bmu*, $79.84. *bmv*, $36.13. *bnr*, $46.09. *bns*, $73.64. *bnt*, $117.95. *bnu*, $99.80. *bnv*, $32.66. *bor*, $55.31. *bos*, $88.41. *bot*, $141.53. *bou*, $119.76. *bov*, $39.20. *epr*, $270.26. *eps*, $431.98. *ept*, $691.60. *epu*, $585.20. *epv*, $191.52. *dqr*, $3.28. *dqs*, $5.25. *dqt*, $8.40. *dqu*, $7.11. *dqv*, $2.33. *eqr*, $328.16. *eqs*, $524.55. *eqt*, $839.80. *equ*, $710.60. *eqv*, $232.56. [The above are answers to questions 1–15, 26–40, etc.]

Discount: [Problems refer to previous exercise.] *amr*, present worth, $110.63; discount, $9.37. *bns*, p. w., $372.48; d., $63.02. *cot*, p. w., $957.73; d., $311.27. *dpu*, p. w., $13.81; d., $4.43. *eqr*, p. w., $1617.75; d., $206.25.

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V. 1

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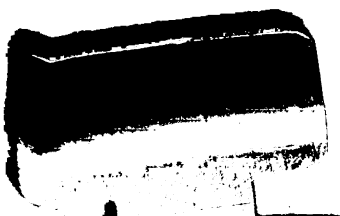

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